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Multi-variable intelligent matching pursuit algorithm using prior knowledge for image reconstruction by l_0 minimization

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A B S T R A C T

Image reconstruction by l_0 minimization is an NP-hard problem that requires exhaustively listing all possibilities of the original signal with a very high computational complexity, which is difficult to be achieved by traditional algorithms. Although greedy algorithm aims at solving l_0 minimization, it is more likely to fall into a suboptimal solution. In this paper, we propose a multi-variable intelligent matching pursuit algorithm (MIMP), which can solve l_0 minimization problem essentially by taking the advantage of the intelligent optimization algorithm in solving combinatorial optimization problems and searching for the global optimal solution to improve the performance of image reconstruction. The updating mechanism of MIMP is designed by introducing the matching strategies of greedy algorithm to accelerate the reconstruction speed. Also, the multi-variable scheme is utilized to sample images and then the joint reconstruction is implemented to the measurements, which can not only improve the reconstruction accuracy but also reduce the computational complexity. Moreover, the edge saliency can be obtained as the prior knowledge to guide the compressive sensing reconstruction, which contributes a lot to reduce the computational complexity and accelerate the reconstruction speed. As the sparsity level of image is hard to be estimated, a new optimization function is proposed to solve this problem without knowing the sparsity level as a prior. Compared with other state-of-the-art algorithms, the proposed method MIMP can achieve a better reconstruction accuracy by solving l_0 minimization essentially with intelligent optimization algorithms. Also, MIMP has a reasonable relatively faster reconstruction speed by introducing the matching strategies of greedy algorithm and using the edge saliency as a prior knowledge. Numerical experiments on several images demonstrate that the proposed method MIMP significantly outperforms the state-of-the-art algorithms and the structure based reconstruction algorithms in PSNR, SSIM and visual quality.

1. Introduction

Compressive sensing (CS) [1–3] theory, proposed by Candes and Donoho in 2006, shows that signal can be reconstructed from many fewer measurements than suggested by Nyquist sampling theory if the signal is sparse or compressive in some domains. If the original signal $x \in R^{N*1}$ is compressive, it can be represented sparsely in the sparse basis ψ. The measurement signal $y \in R^{M*1}$ $(M < N)$ is obtained by $y = \phi * x$, where ϕ is uncorrelated to ψ and satisfies restrict isometry property (RIP). The relations can be represented by the following equation

$$y = \phi * x = \phi * \psi * s \tag{1}$$

where s is the sparse representation of x in the sparse basis ψ.

To simplify Eq. (1), x is assumed to be sparse in the following sections. As we all know, reconstruction is a process of recovering high-dimensional signal x from low-dimensional signal y, which can be obtained by optimizing a constrained optimization problem of sparsity. Then the constrained optimization problem of sparsity can be implemented by the l_0 minimization problem, which can be described in Eq. (2).

$$x = \arg\min \|x\|_0 \quad \text{subject to} \quad y = \phi * x, \tag{2}$$

where, x represents the l_0 norm of x, which is defined by the number of nonzero elements of x. Eq. (2) can be also written in the following form

$$x = \arg\min \|x\|_0 + \lambda * \|y - \phi * x\|_2^2, \tag{3}$$

where λ is a positive constant between 0 and 1, which used to balance the relative weight between the sparsity and the data fidelity item.

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However, l_0 minimization is an NP-hard problem that requires exhaustively listing all possibilities of the original signal and is difficult to be achieved by traditional algorithms. Since l_1 norm [4] is the optimal convex approximation of l_0 norm and it has been proved that l_1 norm is equivalent to the l_0 norm for many problems. It is widely adopted that the l_0 minimization problem can be solved by relaxing it to the l_1 minimization problem [5–7] which is shown in Eq. (4). Nevertheless, a fact that is often neglected is, the conditions guaranteeing the equivalence of l_0 minimization and l_1 minimization are not necessarily satisfied, which leads to a suboptimal solution. Many reconstruction methods have been already proposed recently, such as gradient projection sparse reconstruction [8,9], non-linear filter [10] and iterative threshold [11], which also transform l_0 minimization to other mathematical models and are more likely to fall into suboptimal solutions.

$$x = \arg\min \|x\|_1 + \lambda * \|y - \phi * x\|_2^2. \tag{4}$$

The methods which aims at solving l_0 minimization problem available for compressive sensing reconstruction fall into two categories: greedy algorithm [12–14] and heuristic method. The objective of greedy algorithm is to estimate the support collection and its corresponding coefficients by iterating the coefficients based on the residual correlation. However, the residual correlation can only reflect the correlation degree between coefficients and the measurement signal, but cannot indicate that the bigger the residual correlation is, the bigger the nonzero coefficient is. Also, greedy algorithm is more likely to find a suboptimal solution and cannot solve l_0 minimization essentially. Intelligent optimization algorithm [15,16] is a class of heuristic optimization methods which is famous for its global searching ability and superior performance in solving combinatorial optimization problems. It is natural to consider that intelligent optimization algorithm can be used to solve l_0 minimization problem. Two methods [17,18] based on simulated annealing algorithm are proposed for CS reconstruction by l_0 minimization. However, the single-cycle local searching scheme in [17,18] is hard to find the global optimal solution. Also, the precondition of [17,18] is that the sparsity level must be known, which is a very strong limitation as the sparsity level is always unknown in practice. In our precious work [19,20], we use genetic algorithm and artificial immune algorithm [21,22] to solve l_0 minimization directly. However, due to the randomness of intelligent optimization algorithm, the computational complexity of the proposed methods in [19,20] is high and the reconstruction speed of them slows down.

Image reconstruction [23–25], which is a fundamental problem in the field of image processing, aims to reconstruct the original image with high quality from the degraded measurements. Prior knowledge [26,27] plays an important role in image reconstruction and it has been proved that introducing prior knowledge can improve the efficiency of image reconstruction algorithms. One way of improving the reconstruction quality is to exploit the co-called structured sparsity. Some methods using particular structures such as tree structure [28–30] and dependence structure [31] of the wavelet coefficients have been applied in image reconstruction. Tree-structured correlations of wavelet coefficients is utilized to propose a group-sparsity regulation method in [32]. Bayesian CS reconstruction methods exploiting the prior of the wavelet tree structures have also been developed in [33–35]. Under the wavelet transformation, the main energy of image is concentrated on the wavelet coefficients of the low-frequency subspace and the wavelet coefficients of the high-frequency subspace show high sparsity, which means that the wavelet coefficients corresponding to the edge are significantly different from zero. Suppose the original image is locally smooth except edge area, the locations of the nonzero wavelet coefficients of the high-frequency parts happen to correspond with the edge of the original image. So if the edge information of the image is extracted, it can be applied as prior knowledge to guide the process of image reconstruction, which can reduce the computational complexity. Several representative works based on the guidance of edge information have been proposed, such as edge guided MRI reconstruction [36] and edge based multiple pursuit algorithm (EMPA) [37]. Moreover, a multi-variable scheme using conventional linear sampling and compressive sampling for low frequency coefficients and high frequency coefficients respectively is developed for image reconstruction in [38].

In this paper, we propose a multi-variable intelligent pursuit algorithm (MIMP) using prior knowledge to solve l_0 minimization problem essentially. MIMP aims at searching the support collection and its corresponding coefficients of the original signal intelligently and accurately. As the sparsity level of image is hard to be estimated, a new optimization function is proposed when the sparsity level is unknown in advance. Due to the superior global optimizing strategy of intelligent optimization algorithm, artificial immune algorithm (AIA) is introduced to guide the process of support collection estimation. Artificial immune algorithm performs well in both global searching and local searching, which makes MIMP more likely to find the global optimal solution and have a fast reconstruction speed. The principle of the intelligent estimation is designed by the matching strategies of greedy algorithm, which is beneficial to reduce the computational complexity. Also, based on the high sparsity of the high-frequency parts of wavelet transformation, the multiple measurement scheme, which uses conventional linear sampling for the low-frequency coefficients and compressive sensing for the high-frequency coefficients, is introduced into MIMP to improve the reconstruction quality. In addition, as the edge saliency mostly happens to correspond with the locations of the nonzero wavelet coefficients in high-frequency subspace, it is introduced into MIMP to guide the process of global searching, which can reduce the computational complexity. As MIMP can solve l_0 minimization essentially and by means of the combination of edge saliency and multi-variable joint reconstruction, the proposed method MIMP significantly improves the reconstruction quality of those images with the obvious edges and high sparsity, such as computed tomography (CT) images and magnetic resonance (MR) images. Experiments on image reconstruction demonstrate that the proposed method MIMP produces more superior reconstruction performance than other state-of-the-art algorithms, such as multi-variable OMP (M-OMP), multi-variable Polytope Faces Pursuit algorithm (M-PFP) [39], multi-variable backtracking-based matching pursuit (M-BAOMP) [40], multi-variable compressive sampling orthogonal matching pursuit (M-CoSaOMP) [41], multi-variable Sparsity Adaptive Matching Pursuit algorithm (M-SAMP) [42], tree-based CoSaMP (Tree-CoSaMP) [43], tree-structured wavelet compressive sensing (TSWCS) [33] and edge based multiple pursuit algorithm (EMPA), edge based compressive sensing algorithm (EdgeCS), multi-variable pursuit algorithm using multi-variable K distribution (MPA-MK) and multi-variable pursuit algorithm using multi-variable Laplace distribution (MPA-ML).

The major contributions of this paper are fourfold:

1. MIMP improves the reconstruction quality significantly by taking the advantage of intelligent optimization algorithm in solving l_0 minimization essentially and accelerates the reconstruction speed by introducing matching strategies of greedy algorithm.
2. As the sparsity level of image is hard to be estimated, a new optimization function is proposed for CS reconstruction when the sparsity level is unknown in advance.