Information Sciences 378 (2017) 1–25

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Generalized dominance rough set models for the dominance intuitionistic fuzzy information systems☆

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ARTICLE INFO

Article history:
Received 9 October 2015
Revised 2 September 2016
Accepted 22 October 2016
Available online 24 October 2016

Keywords:
Dominance-based rough set approach (DRSA)
Generalized dominance rough set
Generalized β-dominance rough set
Attribute reduction

ABSTRACT

Dominance rough sets generalize classical rough sets by replacing the equivalence relation with a dominance relation. However, the existing dominance relations are still too restrictive to convenient for its practical applications because they are always considering a strict decreasing or increasing order for each attribute. In fact, in many real-world problems, we only need to employ the decreasing or increasing order in terms of partial attributes rather than all attributes or even just focus on overall evaluations of objects. Based on this phenomenon, in this paper we define two new dominance relations and obtain two generalized dominance rough set models according to define the overall evaluations and add particular requirements for some individual attributes. Meanwhile, the attribute reductions of dominance intuitionistic fuzzy decision information systems are also examined with these two models. Firstly, we define a generalized dominance relation and obtain a generalized dominance rough set model by using the intuitionistic fuzzy additive operator to aggregate individual attribute value of each object into a overall evaluation. The attribute reduction of intuitionistic fuzzy information systems with generalized dominance rough set are also explored. Secondly, we introduce another dominance relation named as generalized β-dominance relation and the generalized β-dominance rough set model by adding a parameter $\beta \in [0, 1]$ in generalized dominance relation in order to control the number of attributes which satisfy dominance relations, from which we can induce all “at least” and “at most” decision rules. An algorithm is developed to compute the lower and upper β-dominance approximations in order to get all the lower and upper reducts of intuitionistic fuzzy decision systems. Some numerical examples are employed to substantiate the conceptual arguments.

1. Introduction

The concept of rough sets was originally proposed by Pawlak [28] in 1982s as a mathematical approach to handle imprecision, vagueness and uncertainty in data analysis [12,27,33–35,42] and has successfully been applied to many practical

☆ This work is supported by National Natural Science Fund of China (71471060), Fundamental Research Funds for the Central Universities Support Program (2015xs71), Macao Science and Technology Development Fund (100/2013/A3).

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http://dx.doi.org/10.1016/j.ins.2016.10.041

problems in machine learning, pattern recognition, decision analysis, process control, knowledge discovery in databases, and expert systems [7,8,10,30–32]. Presently, research on rough sets has formed many significant issues, such as attribute reduction problems [26,37,44], approximation operator models [29,36,41], axiomatic systems [43,46,48], generalizations of rough sets [11,18,39,47] and so on. Among these studies, generalizations of rough sets is an important and meaningful component of rough set theory. Different practical issues can be solved effectively with these generalizations.

Pawlak's rough sets were developed to address those data sets where each object can only take a unique discrete value for every attribute, and furthermore, every attribute in such data sets can induce an equivalence relation on the universe of discourses or a partition of the universe of discourses. However, such an equivalence or partition is still restrictive for many applications because it is not able to discover inconsistencies coming from consideration of criteria, that is, attributes with preference-ordered domains (scales), such as product quality, market share, and debt ratio [18]. To address this issue, many generalized rough set-based approaches were proposed during the past 30 years, among which a very important approach is Dominance-based Rough Set Approach (DRSA) [15–17,19]. This approach is established based on substitution of the equivalence relation by a dominance relation, whereby the ranking properties of criteria can be solved. Therefore, DRSA can be explicitly used to deal with inconsistencies typical to exemplary decisions in multi-criteria decision making problems.

The current processes of DRSA mainly focus on generalizations and developing algorithms for attribute reductions [5,13,14,19–22,25]. DRSA was first proposed in [15] by Greco et al., wherein condition attributes are the criteria and ranked by preference. Thus, the knowledge approximated is a collection of dominance classes, which are sets of objects induced by a dominance relation. For instance, Greco [13] proposed the dominance rough fuzzy set model by introducing DRSA into the fuzzy environment. In this model, the fuzzy target is approximated by using a dominance relation instead of an fuzzy relation. In [14], the classical dominance relation was generalized to be the pairwise compared dominance relation for the analysis of the ordinal properties of preferred degrees of pairs of the objects. Błaszczyński et al. [5,6] proposed the variable consistency dominance-based rough set approach (VC-DRSA) by introducing the concept of variable precision rough set [49] into DRSA, wherein different variants of VC-DRSA are considered. These variants produce more general lower approximations than those computed by DRSA. That is, they defined the lower approximation that certain objects characterized by a strong but not necessarily relation with approximated sets, which achieved by introducing a parameter to control the consistency of the objects included in lower approximations. Hu [21] presented an algorithm to compute the reductions of the variable precision dominance-based rough set model. In [9,22–24,38], the dominance-based rough approximations and their applications were extended in the interval-valued, intuitionistic fuzzy, interval-valued intuitionistic fuzzy information system in which the objects are characterized by imprecise evaluations.

The dominance relation of above work about dominance rough sets are all constructed under the idea of classical dominance relation, which is a strong preference relation on universe (often called outranking) representing a preference on the set of objects with respect to each criterion. However, such a dominance relation still restrictive because it requires the objects satisfy the ranking properties with respect to each criterion of information systems rather than partial criteria. Thus, in the process of constructing dominance classes by using classical dominance relation, even if a single attribute of an object is inferior to that of another one, it will be excluded from the dominance class of the second object. This property can be important especially in case of multi-criteria decision making problems of large universes.

As well known, in many group decision problems, we sometimes just need to find the objects with best synthetical evaluation and ignore the individual attribute values. In other words, although one object has an inferior value in some individual attributes, as far as its synthetical evaluation is better than that of others, it also can be chose to the optimal candidates. On the other hand, in many real-world problems, not only do we need to find the objects with best synthetical evaluations, but also require that the object is better than others in some individual attributes. For example, for a computer audit risk assessment, if we want to find some objects dominate audit object x with respect to overall evaluation, then we only need to compute the overall evaluation of all the objects and to choose those objects better than that of x. While in special situations, not only do we need to find the objects with overall evaluation superior than x, but also require that the object is better than x in partial individual attributes, thus it is necessary to introduce a parameter $\beta(0 < \beta \leq 1)$ to control the number of criteria of one object better than others. Based on these two considerations, we define two new dominance relations by taking into account the overall evaluations of objects and ranking objects in terms of partial criteria rather than each criterion in intuitionistic fuzzy information systems, respectively. The overall evaluations of objects are obtained by using the intuitionistic operation to fuse all attribute values into an intuitionistic value. Firstly, one can obtain a new dominance relation by using the overall evaluations of objects to rank the objects, so called generalized dominance relation, which is used to describe the set of objects with better overall evaluations. Secondly, the generalized β-dominance relation is defined based on dominance intuitionistic fuzzy information systems by taking a parameter β in generalized dominance relation. Up to now, two new dominance relations are obtained according to the decision maker's requirements on " overall evaluations" and " individual attribute values", respectively. By employing the generalized dominance relation into rough set model, the notion of generalized dominance rough set model with dominance intuitionistic fuzzy information system is introduced. We also investigate the attribute reduction of dominance intuitionistic fuzzy information systems under the framework of generalized dominance rough set model. In addition, we develop the generalized β-dominance rough set model by incorporating the generalized β-dominance relation, from which one can induce all "at least" and "at most" decision rules. An algorithm is explored to obtain the lower and upper generalized β-dominance approximations in order to acquire attribute reduction of dominance intuitionistic fuzzy information systems. Some numerical examples are employed to substantiate the conceptual arguments. It is worth pointing out that the two models proposed in this paper are