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Finite-time H_∞ state estimation for switched neural networks with time-varying delays

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ABSTRACT

This study considers the problem of finite-time H_∞ state estimation for the switched neural networks with time varying delay, based on the theories of the switched systems. Sufficient conditions for the switched neural networks to be finite time stable and finite time bounded are derived. These conditions are delay dependent and are given in terms of linear matrix inequalities (LMIs). Average dwell time of switching signals is also given such that switched neural networks are finite-time stable or finite-time bounded. By resorting to the average dwell time approach and Lyapunov–Krasovskii functional technology, the H_∞ estimator design are developed in terms of solvability of a set of linear matrix inequalities. Finally, numerical examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

During the last few decades, neural networks have received great attention due to their extensive applications in various fields such as signal processing, pattern recognition, fixed-point computation and other scientific areas. Many papers have focused on studying the existence, uniqueness, and global robust asymptotic stability of the equilibrium point in the presence of time delays and parameter uncertainties for various classes of nonlinear neural networks (see [1–6]). Time delays are inevitable in the implementation of artificial neural networks as a result of the finite switching speed of amplifier. Time delays are very common in various practical systems (see [7]). However, the presence of a delay may lead to instability and the poor performance of control systems, as well as making the analysis of problems for switched time-varying delays systems much more complicated [8–10].

Switched neural networks (SNNs) arise in various fields of real life world, such as manufacturing, communication networks, autopilot design, automotive engine control, computer synchronization, traffic control, and chemical processes etc., [11–14]. In the past few decades, increasing attention has been paid to the analysis and synthesis of SNNs due to their significance in both theory and applications, and many significant results have been obtained for the analysis and design of SNNs, (see [15–19]). For the SNNs, however, most existing results have cursorily addressed all the subsystems being unstable with a rising upper bound under the Lyapunov-like framework (see [20,21]). Indeed, the H_∞ performance analysis problem for SNNs with any category of switching signals has always been the most intense research topic in this area [22–24].

Dwell-time switching supervisors force every candidate controller to remain in the loop for, at least, τ_a its of time, thus guaranteeing a fixed dwell-time of τ_0. Unfortunately, with nonlinear system this may lead to finite escape of the closed-loop (see [25–27]). Average Dwell Time (ADT) switching is a class of restricted switching signals which means that the number of switches in a finite interval is bounded and the average dwell time between consecutive switching is not less than a constant. It was well known that the ADT scheme characterizes a large class of stable switching signals than dwell time scheme, and its extreme case is the arbitrary switching. Thus, the ADT method is very important not only in theory, but also in practice, and considerable attention has been paid, and a lot of efforts have been done to take advantage of the ADT method to investigate the stability and stabilization problems both in linear and nonlinear systems (see [28–32]). The concept of “dwell time” is extended to the concept of ”average dwell time”. It has been recognized that ADT is more flexible and efficient in system stability analysis, since the ADT switching strategy may contain signals that occasionally have

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consecutive discontinuities separated by less than a constant τ_a (it should be compensated by switching sufficiently slowly later). Correspondingly, the stability analysis and control synthesis for the switched systems with ADT have been also reported in both the continuous-time and discrete-time domains. However, the property in the ADT switching that the average time interval between any two consecutive switching is at least τ_a, which is independent of the system modes, is probably still not anticipated. Besides, it has been well shown in the literature that, the minimum of admissible ADT is computed by two mode-independent parameters, i.e., the increase coefficient of the Lyapunov-like function at switching instants and the decay rate of the Lyapunov-like function during the running time of subsystems. It is straightforward that such a setup of the two common parameters for all subsystems in a mode-independent manner will give rise to a certain conservativeness. Therefore, to extend the existing studies on the switched systems by providing two mode-dependent parameters, which will lead to a mode-dependent ADT accordingly, is worthwhile to proceed, which inspires us for this study. (see [33–35] and refer there in).

It is well known that classical control theory mainly tackles the asymptotic behavior of the system trajectories over an infinite interval. In practice, however, main attention may be related to the behavior of the dynamics over a fixed finite time interval, such as keeping the acceptable values in a prescribed bound in the presence of saturations [36,37]. To achieve faster convergence rate in time delay neural networks, an effective method is to use finite-time boundedness techniques [38–40]. Finite-time boundedness means the optimality in convergence time. Moreover, the finite-time control techniques have demonstrated better robustness and disturbance rejection properties. And thus far, there is few published papers considering finite-time bounded of neural networks [41–46].

One of the important and interesting problems in the analysis of finite-time switched neural networks is their stability. In the implementation of networks, time delays exist due to the finite switching speed of amplifiers and transmission of signals in the network community, which may lead to oscillation, chaos and instability [47–49].

One of the most effective methods for the state estimation problem is to design a filter, which has been concerned for switched delay systems with different performance indexes such as Kalman filtering, L_2-L_∞ filtering, H_2 filtering, and H_∞ filtering. Compared with others, H_∞ filtering permits the exogenous noises to be arbitrary with bounded energy or average power and without known precise statistics. In the H_∞ filtering setting, a state estimator is designed to guarantee that the filter error system is stable in the absence of the external noise signals and its H_∞ performance from the external noise signals to the estimation error is below a prescribed level of noise attenuation studied in [50–52]. For some representative works on H_∞ filtering for switched delay systems, to name a few, investigated an exponential H_∞ filtering for continuous-time switched systems with interval time-varying delay to assure the exponential stability with a weighted H_∞ performance for the filtering error system via the free-weighting matrix technique. A finite-time H_∞ filter design procedure was developed by using the matlab toolbox for continuous-time switched time-varying delay systems [53–56].

H_∞ control theory considers the worst case of external disturbances to design an optimal controller to achieve the desired performance. More recently, H_∞ control theory has been applied to an actual building in Tokyo, Japan using a pair of mass dampers to reduce the bending-torsion motion due to earthquakes. Further, a liquid mono-propellant rocket motor with a pressure feeding system has been considered as a numerical design example in [57]. In the H_∞ control technique, the main design goal is to force the gain from un-modeled dynamics, external disturbances, and approximation errors to be equal or less than a prescribed disturbance attenuation level (H_∞ attenuation constraint). This goal is generally represented as an LMI problem. In the time-domain approach, the direct Lyapunov method is a powerful tool for studying the problem of stability and H_∞ control for systems with delay. Thus, a great number of delay-dependent stability and stabilization results for H_∞ controller design have been reported, and many effective results have been provided to reduce the conservatism of stability results for further reducing the disturbance attenuation level [58–60].

Motivated by the above-mentioned discussion, in this paper finite-time boundedness by constructing a Lyapunov functionals containing triple integral with exponential function such as $V_6(\zeta_t, i) = \int_{-\bar{\tau}}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} e^{\alpha_i(t-s)}\zeta^T(s)S_{2i}\zeta(s)ds\,d\nu\,d\theta, V_7(\zeta_t, i) = \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} e^{\alpha_i(t-s)} f^T(\zeta(s))Y_{1i}f(\zeta(s))ds\,d\theta$, which play a key role in obtaining more less conservative stability conditions. By utilizing convex combination and integral inequality lemmas the finite time boundedness conditions are derived. To the best of authors knowledge, triple integral functional and convex combination method have not been utilized for the study of finite time control of switched neural networks with time-varying delays. The optimal finite-time observer is designed to minimize the bound of error state by an linear matrix inequality (LMI) based iterative algorithm using the average dwell time method and an integral inequality; then, the H_∞ performance is taken into account and finite-time H_∞ state estimation are developed.

Notation: The notation used in this paper is standard. R^n denotes n dimensional Euclidean, the superscript "T" denotes the transpose and the notation $P>0$ (≥ 0) means P is real symmetric positive (semi-positive) definite, max(P) and min(P) denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. diag$\{a_i\}$ denotes the diagonal matrix with the diagonal elements a_i, $(i=1,2,\ldots)$. The asterisk $*$ in a matrix is used to denote a term that is induced by symmetry.

2. System description and preliminaries

Consider the following switched neural networks with time varying delay

$$\left.\begin{aligned} \dot{x}(t) &= -A_{\sigma(t)}x(t)+B_{\sigma(t)}f(x(t))+B_{d\sigma(t)}f(x(t-\tau(t)))+D_{1\sigma(t)}w(t),\\ y(t) &= C_{\sigma(t)}x(t)+G_{\sigma(t)}f(x(t))+D_{2\sigma(t)}w(t),\\ z(t) &= E_{\sigma(t)}x(t)+D_{3\sigma(t)}w(t),\\ x(t) &= \phi(t), t\in[-\bar{\tau},0). \end{aligned}\right\} \tag{1}$$

where $x(t)\in R^n$ is the state, $w(t)\in R^m$ is the disturbance input which belongs to $L_2[0,\infty)$, $y(t)\in R^q$ is the measured output, $z(t)\in R^p$ is the signal that needs to be estimated, and $A_{\sigma(t)}$, $B_{\sigma(t)}$, $B_{d\sigma(t)}$, $C_{\sigma(t)}$, $G_{\sigma(t)}$, $E_{\sigma(t)}$, $D_{1\sigma(t)}$, $D_{2\sigma(t)}$, $D_{3\sigma(t)}$ are known real constant matrices of appropriate dimensions.

$f(x(t)) = [f_1(x(t)), (f_2(x(t)), (f_3(x(t)), \ldots, (f_n(x(t))]^T$ denotes the neuron activation function.

In [2] the prototypical architecture for such switched neural networks is shown in Fig. 1. $\tau(t)$ is the differentiable mode-dependent time-varying function, which satisfies

$$0\leq \tau(t)\leq \bar{\tau}, \tag{2}$$

$$h_0\leq \dot{\tau}(t)\leq h_m, \tag{3}$$

where $\bar{\tau}$, h_0, and h_m are constant scalars (Fig. 2).