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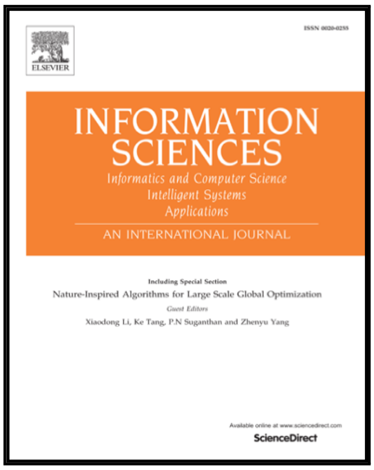


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Giampiero Chiaselotti, Davide Ciucci, Tommaso Gentile,
Federico G. Infusino

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# The Granular Partition Lattice of an Information Table

Giampiero Chiaselotti[o] and Davide Ciucci[*] and Tommaso Gentile[o] and Federico G. Infusino[o]

[*] *Department of Informatics, Systems and Communications, University of Milano-Bicocca, 20126 Milano, Italy*
[o] *Department of Mathematics and Informatics, University of Calabria, Via Pietro Bucci, Cubo 30B, 87036 Arcavacata di Rende (CS), Italy.*

**Abstract**

In this paper we study the lattice of all indiscernibility partitions induced from attribute subsets of a knowledge representation system (information table in the finite case). This lattice, that we here call granular partition lattice, is a very well studied order structure in granular computing and data base theory and it provides a complete hierarchical classification of the knowledge obtained from all possible choices of attribute subsets. We show that it has a lattice structure also in the infinite case and we provide several isomorphic characterizations for this lattice. We discuss the potentiality of this order structure from both a micro-granular and a macro-granular perspective. Furthermore, the sub-poset of all the *indiscernibility closures* needed to determine when an arbitrary partition is an indiscernibility one is studied. Finally, we show the monotonic behaviour of the granular partition lattice with respect to entropy of partitions and attribute dependency in decision tables.

*Keywords:* Set Partition, Rough sets, Granular Computing, Entropy

## 1. Introduction

Let us consider a basic structure to represent knowledge: an attribute-value system, i.e., a table whose rows represent the *objects* of a finite universe set $U$ and the columns the *attributes* of another finite set $Att$. Usually, in rough set theory this structure is called *information table* $\mathfrak{I}$ and it corresponds to a relation in first normal form in database theory [18].
If $A$ is an attribute subset of $Att$, we can build the usual rough-set indiscernibility partition $\pi_{\mathfrak{I}}(A)$ of $U$ induced by $A$. The set $\Pi_{ind}(\mathfrak{I})$ of all these indiscernibility partitions is partially ordered by the usual partial order $\preceq$ on the set $\Pi(U)$ of all (not necessarily indiscernibility) set partitions of $U$. The poset $\mathbb{P}_{ind}(\mathfrak{I}) := (\Pi_{ind}(\mathfrak{I}), \preceq)$ is a well known order structure in rough set theory and granular computing [76]. In [31] it has been proved that $\mathbb{P}_{ind}(\mathfrak{I})$ is a complete lattice, but not necessarily a sub-lattice of the lattice $\mathbb{P}(U) := (\Pi(U), \preceq)$.
In this paper we call *indiscernibility partition lattice* the poset $\mathbb{P}_{ind}(\mathfrak{I})$, and we will study this order structure when $U$ and $Att$ are arbitrary sets, i.e. not necessarily finite sets. For this general case we use the term *knowledge representation system* [41, 68], and we deserve the term *information table* to the case when $U$ and $Att$ are both finite sets. The important fact to note here is that, in order to prove the completeness of $\mathbb{P}_{ind}(\mathfrak{I})$ also in the non-finite case, we cannot use a technique similar to that used in [31], therefore we introduce the isomorphic notion of *maximum partitioner poset* for a knowledge representation system $\mathfrak{I}$ and prove it is a complete lattice. More in detail, we identify any indiscernibility partition $\pi \in \Pi_{ind}(\mathfrak{I})$ with the greater attribute subset $A$ such that $\pi_{\mathfrak{I}}(A) = \pi$, and we call such a subset $A$ the *maximum partitioner* of $\pi$. We then introduce the set $MAXP(\mathfrak{I}) := \{Max(\pi) : \pi \in \Pi_{ind}(\mathfrak{I})\}$ and the poset $\mathbb{M}(\mathfrak{I}) := (MAXP(\mathfrak{I}), \subseteq^*)$, where $\subseteq^*$ is the dual set inclusion order between attribute subsets. we will show that $\mathbb{M}(\mathfrak{I})$ is a complete lattice that is isomorphic to the granular partition poset $\mathbb{P}_{ind}(\mathfrak{I})$. As a consequence of this isomorphism, we carry out the structure of complete lattice from $\mathbb{M}(\mathfrak{I})$ towards $\mathbb{P}_{ind}(\mathfrak{I})$ . Given this result, it is clear that the maximum partitioner notion is strictly related to a global view of the knowledge induced by a knowledge representation system and that the introduction of the order structure $\mathbb{M}(\mathfrak{I})$ is fundamental for further detailed investigations of the indiscernibility partition lattice $\mathbb{P}_{ind}(\mathfrak{I})$.

*Email address:*
giampiero.chiaselotti@unical.it,ciucci@disco.unimb.it,gentile@mat.unical.it,f.infusino@mat.unical.it
(Giampiero Chiaselotti[o] and Davide Ciucci[*] and Tommaso Gentile[o] and Federico G. Infusino[o])

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