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Adaptive projective lag synchronization of uncertain complex dynamical networks with disturbance

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ABSTRACT

Projective lag synchronization behavior with non-delay and delay coupling in complex dynamical network model is investigated in this paper. Based on Lyapunov stability theory, adaptive control scheme is applied to achieve the projective lag synchronization model with identical, different nodes and even if the delay coupling have constant time delay or time-varying coupling delay. In addition, the model consists of disturbances and fully unknown parameters. Theses parameters are identified by adaptive control and update law. Finally, the simulation results reveal that the states of the dynamical network with non-delay and delay coupling can be asymptotically synchronized onto a desired scaling under the designed controller. Additionally, numerical examples demonstrate the effectiveness of the proposed method.

1. Introduction

A complex dynamical network consists of coupling nodes where each node is a nonlinear dynamical system connecting with the others via a topology defined on the network edges. In various disciplines, the complex networks have already been shown to exist in many real world systems, such as computer networks, world wide web, telephone call graphs, food webs, neural networks, electrical power grids, etc [1–4]. Thus, studying the characteristics of complex networks and their structurla properties have become an important issue. During the last decades, the control and the synchronization behavior of dynamical systems have been a hot topic. Recently, many kinds of synchronization have been observed in interacting chaotic system [5–15]. Meanwhile, many different effective control schemes have been reported to realize the various kinds of synchronization for complex networks [16–25].

In practical applications, time delay is ubiquitous in many systems due to the finite switching speed of amplifiers, finite signal propagation time in networks, finite reaction times, memory effects and so on [26–28]. In addition, time delay may cause undesirable dynamic behaviors such as oscillation, instability and poor performance. Therefore, it is very important to solve the synchronization problem of complex dynamical networks with time delays. Some researchers have proposed some results in this area. Guo proposed pinning control scheme to achieve lag synchronization of complex dynamical networks [29]. In addition, the pining control has been proposed for achieving hybrid synchronization of the general complex dynamical networks with delayed and non-delayed coupling by Wu and Lu [30]. Based on several nonlinear controllers [31], the problem of projective synchronization with non-delayed and constant delayed coupling in drive-response dynamical networks consisting of identical nodes and different nodes was studied. Based on impulsive control method, Zheng [32,33] studied projective synchronization between general complex networks with coupling time-varying delay and with multiple time-varying delays respectively. In [34] the authors studied projective lag synchronization of the general complex dynamical networks with different nodes via adaptive control. Based on hybrid feedback control, Du et al. [35] proposed a method with function projective synchronization in complex dynamical networks with both cases constant time delay and with time-varying coupling delay. Due to the complexity of the real world, not all of the system parameters are known. Parameter estimations of single or coupled nonlinear systems based on the complete synchronization method have been investigated [36–39].

Motivated by the above discussion, the aim of this paper is to deal with the problem of a projective lag synchronization (PLS) scheme in drive-response dynamical networks (DRDNs) model with non-delay and delay coupling consisting of identical and different nodes. Both the drive and the network nodes have uncertain parameters and disturbance. Based on Lyapunov stability theory, an adaptive control method is designed to achieve the

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projective lag synchronization in DRDNs with constant and time-varying coupling delay. Adopting adaptive gains laws, the unknown parameters are estimated. In addition, the controller is designed to overcome the unknown bounded disturbance. Moreover, numerical simulations are performed to verify the effectiveness of the theoretical results.

The rest of this paper is organized as follows: The network model is introduced in Section 2. A general method of PLS between uncertain complex dynamical networks model with constant coupling delayed by an adaptive control method is discussed in Section 3. Section 4 deals with a further investigation of PLS between uncertain dynamical networks model with time-varying coupling delayed by using the proposed method. Examples and their simulations are shown in Section 5. Finally, the conclusions are presented in Section 6.

2. Model description

Consider a controlled complex dynamical network with delay coupling consisting of N linearly and diffusively different nodes with both uncertain parameters and disturbance described as follows:

$$\dot{x}_i^r(t)=g_i(x_i^r(t))+G_i(x_i^r(t))\theta_i+c\sum_{j=1}^{N}a_{ij}\Gamma x_j^r(t)+c\sum_{j=1}^{N}b_{ij}\Gamma x_j^r(t-d_i)+\Delta_i(t)+u_i(t),\quad i=1,2,\ldots,N, \tag{1}$$

where $x_i^r=(x_{i1}^r,x_{i2}^r,\ldots,x_{in}^r)^T\in\mathbf{R}^n$ denotes the state vector of the ith node, $g_i:\mathbf{R}^n\longrightarrow\mathbf{R}^n$ and $G_i:\mathbf{R}^n\longrightarrow\mathbf{R}^{n\times m_i}$ are the known continuous nonlinear function matrices determining the dynamic behavior of the node, θ_i is the unknown constant parameter vector, $u_i\in\mathbf{R}^n$ is the control input, c is the coupling strength, and $d_i\ge 0$ is an unknown coupling delay. Here $\Gamma=diag(\gamma_1,\gamma_2,\ldots,\gamma_n)$ is the inner coupling matrix with $\gamma_i=1$ for the ith state variable, i.e. matrix Γ determines the variables with which the nodes in the system are coupled. $A=(a_{ij})\in\mathbf{R}^{N\times N}$ and $B=(b_{ij})\in\mathbf{R}^{N\times N}$ are the non-delay and delay coupling matrixes representing the topological structure of the networks, where a_{ij} and b_{ij} are defined as follows: If there is a connection from node i to node $j(j\ne i)$, then the coupling $a_{ij}\ne 0, b_{ij}\ne 0$; otherwise, $a_{ij}=b_{ij}=0(j\ne i)$, and the diagonal elements of matrixes A,B are defined as

$$a_{ii}=-\sum_{j=1,j\ne i}^{N}a_{ij},\quad b_{ii}=-\sum_{j=1,j\ne i}^{N}b_{ij}\quad i=1,2,\ldots,N. \tag{2}$$

The reference node is described as follows:

$$\dot{x}^d(t)=f(x^d(t))+F\left(x^d(t)\right)\Phi+\Delta_d(t), \tag{3}$$

where the superscripts d stand for the drive system $x^d=(x_1^d,x_2^d,\ldots,x_n^d)^T\in\mathbf{R}^n$ denotes the state vector of the drive system, $f:\mathbf{R}^n\longrightarrow\mathbf{R}^n$, and $G_i:\mathbf{R}^n\longrightarrow\mathbf{R}^{n\times m_i}$ are the known continuous nonlinear function matrices determining the dynamic behavior of the node; Φ is the unknown constant parameter vector, and Δ_d contains the mismatched terms.

The projective lag synchronization error is defined as

$$e_i(t)=x_i^r(t)-\alpha x^d(t-\tau),\quad i=1,\ldots,N \tag{4}$$

where α is the nonzero scaling factor, $\tau>0$ is a constant representing time delay or lag. Then the objective of this paper is to design a controller $u_i(t)$ such that the reference nodes (1) and dynamical networks (3) are asymptotically synchronized such that

$$\lim_{t\longrightarrow\infty}\|x_i^r(t)-\alpha x^d(t-\tau)\|=0,\quad i=1,\ldots,N \tag{5}$$

which means that the network (1) is projective lag synchronized with reference node (3).

The error dynamics is obtained:

$$\dot{e}_i(t)=g_i(x_i^r(t))+G_i(x_i^r(t))\theta_i+c\sum_{j=1}^{N}a_{ij}\Gamma e_j(t)+c\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-d_i)+u_i(t)+\Delta_i(t)-\alpha\Big(f(x^d(t-\tau))+F(x^d(t-\tau))\Phi+\Delta_d(t-\tau)\Big) \tag{6}$$

Assumption 1 (*[37]*). For any positive constant ε_i the time-varying disturbance $\Delta_i(t)$ is bounded i.e. $\|\Delta_i(t)\|\le\varepsilon_i$

3. PLS in DRDNs with constant delay

In this section, we design an adaptive control method to realize projective lag synchronization for uncertain complex dynamical networks with constant delay coupling.

Theorem 3.1. *The projective lag synchronization error (6) is asymptotically stable with a given time delay τ and scaling factor α, by using the following control input and adaptive laws:*

$$u_i(t)=-q_ie_i(t)-\beta_isgn(e_i(t))-g_i(x_i^r(t))-G_i(x_i^r(t))\hat{\theta}_i(t)+\alpha\Big(f(x^d(t-\tau))+F(x^d(t-\tau))\hat{\Phi}(t)\Big),\quad i=1,\ldots,N, \tag{7}$$

$$\dot{\hat{\theta}}_i(t)=k_1G_i^T(x_i^r(t))e_i(t), \tag{8}$$

$$\dot{\hat{\Phi}}(t)=-k_2F_i^T(x_i^d(t-\tau))e_i(t), \tag{9}$$

$$\dot{q}_i(t)=k_3e_i^T(t)e_i(t), \tag{10}$$

$$\dot{\beta}_i(t)=k_4e_i^T(t)sgn(e_i(t)), \tag{11}$$

where k_1,k_2,k_3, and k_4 are positive constants and $\hat{\Phi}(t)$ and $\hat{\theta}_i(t)$ are the estimated parameters for the reference node (1) and network (3), respectively.

Proof. Construct the Lyapunov function candidate as follows:

$$V(t)=\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t)+\frac{1}{2k_1}\sum_{i=1}^{N}\tilde{\theta}_i^T(t)\tilde{\theta}_i(t)+\frac{1}{2k_2}\sum_{i=1}^{N}\tilde{\Phi}_i^T(t)\tilde{\Phi}_i(t)+\frac{1}{2k_3}\sum_{i=1}^{N}\tilde{q}_i^2(t)+\frac{1}{2k_4}\sum_{i=1}^{N}\tilde{\beta}_i^2(t)+\frac{1}{2}\int_{t-d}^{t}\sum_{i=1}^{N}e_i^T(s)e_i(s)ds, \tag{12}$$

where $\tilde{\Phi}_i(t)=\hat{\Phi}_i(t)-\Phi,\tilde{\theta}_i(t)=\hat{\theta}_i(t)-\theta_i,\tilde{q}_i(t)=q_i(t)-q_i^*,\tilde{\beta}_i(t)=\beta_i(t)-\beta_i^*$, where q_i^* and β_i^* are positive constants.

The time derivative of $V(t)$ along the error dynamics (6) is

$$\dot{V}(t)=\sum_{i=1}^{N}\left[e_i^T(t)\dot{e}_i(t)+\frac{1}{k_1}\dot{\hat{\theta}}_i^T(t)\tilde{\theta}_i(t)+\frac{1}{k_2}\dot{\hat{\Phi}}_i^T(t)\tilde{\Phi}_i(t)+\frac{1}{k_3}\dot{q}_i\tilde{q}_i(t)+\frac{1}{k_4}\dot{\beta}_i\tilde{\beta}_i(t)\right]+\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{1}{2}\sum_{i=1}^{N}e_i^T(t-d)e_i(t-d) \tag{13}$$

By application of the control input (7)to the error dynamics $\dot{e}_i(t)$ we have

$$\dot{V}(t)=\sum_{i=1}^{N}\left[e_i^T(t)\Big(-q_ie_i(t)-\beta_i(t)sgn(e_i(t))-G_i(x_i^r(t))\tilde{\theta}_i(t)+\alpha F(x^d(t-\tau))\tilde{\Phi}(t)\Big)\right]+\sum_{i=1}^{N}\left[e_i^T(t)\left(c\sum_{j=1}^{N}a_{ij}\Gamma e_j(t)+c\sum_{j=1}^{N}b_{ij}\Gamma e_j(t-d)+\Delta_i(t)-\alpha\Delta_d(t-\tau)\right)\right]+\sum_{i=1}^{N}\left[\frac{1}{k_1}\dot{\hat{\theta}}_i^T(t)\tilde{\theta}_i(t)+\frac{1}{k_2}\dot{\hat{\Phi}}_i^T(t)\tilde{\Phi}_i(t)+\frac{1}{k_3}\dot{q}_i\tilde{q}_i(t)+\frac{1}{k_4}\dot{\beta}_i\tilde{\beta}_i(t)\right]+\frac{1}{2}\sum_{i=1}^{N}e_i^T(t)e_i(t)-\frac{1}{2}\sum_{i=1}^{N}e_i^T(t-d)e_i(t-d) \tag{14}$$