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Distributed H_∞ consensus tracking control for multi-agent networks with switching directed topologies

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ABSTRACT

This paper is concerned with the H_∞ consensus tracking control problem for linear leader–follower systems with switching directed networks and exogenous disturbances, where the condition of zero control input for leader node is not assumed. Suppose the interaction network of all subsystems may switch among finite digraphs, which contains some non-identical directed spanning trees. Based only on the state information of neighboring agents, a design procedure for constructing the distributed consensus protocol is developed. By employing the topology-dependent multiple Lyapunov functions (MLFs) method and algebraic graph theory, the criteria of consensus protocol design is given in terms of linear matrix inequalities (LMIs). It is proved that the H_∞ consensus tracking problem for multi-agent systems (MASs) under dynamic directed topologies can be solvable if the topology average dwell time satisfies a certain switching condition. Finally, a network system of F-18 aircraft is given as an example to verify the effectiveness of the proposed design method.

1. Introduction

As a fundamental problem in the coordination control of agent networks, the integration of communication and control in distributed architectures has motivated the recent developments of consensus theory, see, e.g., [1–4], and so on. With the fast development in the automatic agents in cooperative and distributed environments, e.g., mobile robots, cooperating UAV team operations (surveillance and reconnaissance), formation flying of UAVs and satellites, and vehicle platoons (see [5–8]), to achieve a coordination goal, each practical subsystem must exchange the local information to its neighbors via communication networks. In general, according to whether the consensus values are prescribed common trajectories, the coordination for MASs can be categorized into leaderless consensus problem (or cooperative regulator problem) and leader-following consensus problem (or synchronization/consensus tracking problem). In both the leaderless and leader–follower scenarios, the interconnection/communication topology plays a important role to guarantee the information flow can be transmitted among the whole agent networks [9–13].

For practical implementation of coordination control for MASs, many uncertain infectors including communication failures or creations, environmental constraints, limitations of sensor hardware, and so on, may lead to communication deterioration of the interaction network. Thus, in order to improve the reliability of networked coordination, it is scientifically meaningful to investigate the consensus (synchronization) problem of agent networks subject to time-varying communication topology. In [2,9], the distributed tracking problems of first-order and second-order MASs with switching networks are considered. In [14], leader-following consensus is addressed for networked agents with linear node dynamics when the undirected underlying topology that is jointly connected evolves with time. Very recently, for the switching directed topologies case, the consensus tracking control problems for linear agent networks are further investigated in [15–17] by means of algebraic graph theory and *M*-matrix properties.

However, it should be pointed out that the works in [14–17] are commonly required that the leader's control input is assumed to be zero. As a matter of fact, this assumption is usually unrealistic in many practical leader-following consensus. Although the distributed adaptive schemes in [18–22] can deal with the consensus tracking against a dynamic leader with nonzero input, these have the restriction that the underlying network is an undirected connected graph with a fixed topology. In addition, the conventional techniques to handle the unknown signal for centralised systems, such as adaptive fuzzy methods [23–25], sliding mode control [26–28], cannot be directly applied in a distributed architecture. To the best of our knowledge, the cooperative tracking problem have not been solved under switching directed topologies with leader's

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time-varying input. The crucial obstacles in the study are summarized into two parts: Firstly, the local controller gain design cannot be decoupled from the directed topology structure; Secondly, it is very difficult to establish convergence analysis of global tracking errors mainly due to the nonzero input cannot be eliminated completely under the switching directed networks. Therefore, it is necessary and also more challenging to overcome these obstacles, which motivates the current investigation.

On the other hand, exogenous disturbances are always inevitable in most of practical environment. Thus, how to improve disturbance rejection performance and guarantee consensus tracking for agent networks become a meaningful issue. In [29], the H_∞ consensus protocol for a team of first-order systems is presented under digraphs. Distributed H_∞ consensus problems are further investigated in [30–32] for higher-order MASs via relative information of neighbors under fixed communication networks. Recently, some distributed H_∞ control schemes for linear agent networks under switching digraphs have been proposed in [33–37] based on multiple Lyapunov functions method and algebraic graph theory. It is worth mentioning that the aforementioned works focus on the H_∞ consensus coordination without leader dynamics. However, there has been relatively little work for leader-following coordination with disturbance rejection performance and time-varying directed networks up to now. This is mainly due to the unknown signals including nonzero leader's input and exogenous disturbances have effects on the networks synchronization simultaneously as well as the complexity arising from interaction between the cooperative disturbance rejection design and the non-symmetry dynamic network. Hence, the study of H_∞ consensus tracking control of MASs with switching directed networks is of great significance and remains an open area.

Motivated by the above discussions, this paper investigates the distributed H_∞ tracking control problem for high-order leader–follower systems under switching directed topologies. Compared with the existing results on H_∞ consensus tracking control problem of MASs, no assumption on zero input of the leader is made in the current study. The main contributions of this paper are as follows:

1. Inspired by the works of [15,16,34], where the topology dwell time methods are employed to handle multiagent consensus under switching interaction networks, a novel cooperative H_∞ tracking controller design is presented to achieve the leader-following coordination.
2. To overcome the main obstacle, by utilizing structure properties of directed Laplacian matrix, the nonzero leader's input can be eliminated completely by designing auxiliary control gain via only neighbourhood state information.
3. By making use of the MLFs method and algebraic graph theory, a sufficient condition for the consensus protocol design is given. It is shown that the proposed consensus algorithm can guarantee that the H_∞ consensus tracking problem is solvable in the presence of interconnection network switching and external disturbances.

This paper is organized as follows. Section 2 provides the preliminaries and problem statement. The main results of the paper are presented in Section 3. In Section 4, an illustrative example is established. Finally, conclusions are drawn in Section 5.

Notation: $\mathbb{R}^{m\times n}$ is the set of $m\times n$ real matrices. The superscript T means transpose for real matrices. I_n is the identity matrix of dimension n. . All matrices, if not explicitly stated, are supposed to have compatible dimensions. diag$\{D_1,\dots,D_n\}$ is a block-diagonal matrix with matrices D_i, $i=1,\dots,n$, on its diagonal. The matrix inequality $F\le G$ means that F and G are symmetry matrixes and that $F-G$ is semi-negative definite. For matrix $F\in\mathbb{R}^{n\times n}$, $\lambda_{max}(F)$ and $\lambda_{min}(F)$ are its maximum eigenvalue and the minimum eigenvalue respectively. $F\otimes G$ denotes the Kronecker product of two matrices $F\in\mathbb{R}^{m\times n}$ and $G\in\mathbb{R}^{p\times q}$. $0_{m\times n}\in\mathbb{R}^{m\times n}$ is the matrix with all entries to be zero and $\mathbf{1}_{m\times n}\in\mathbb{R}^{m\times n}$ represents the matrix with all entries to be one.

2. Preliminaries and problem statement

2.1. Basic graph theory

A weighted digraph (directed graph) $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ is defined by a nonempty finite set of $\mathbf{N}$ nodes $\mathcal{V}=\{v_1,\dots,v_{\mathbf{N}}\}$, a set of edges $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ and an associated adjacency matrix $\mathcal{A}=(a_{ij})\in\mathbb{R}^{\mathbf{N}\times\mathbf{N}}$. Define $\mathcal{N}=\{1,\dots,\mathbf{N}\}$. An edge rooted at node j and ended at node i is denoted by (v_j,v_i), which means information can flows from node j to node i and vice versa. a_{ij} is the weight of edge (v_j,v_i) and $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, otherwise $a_{ij}=0$. A path from node i to node j is defined by a sequence of successive edges in the form $\{(v_i,v_k),(v_k,v_l),\dots,(v_m,v_j)\}$. A loop in a graph is a sequence of paths starting and ending at the same node. It is assumed that there are no repeated edges and no self loops, i.e., $a_{ii}=0$, for all $i\in\mathcal{N}$. Node j is called a neighbor of node i if $(v_j,v_i)\in\mathcal{E}$. The set of neighbor of node i is defined by $N_i=\{j|(v_j,v_i)\in\mathcal{E}\}$. The degree matrix $\mathcal{D}$ is defined by $\mathcal{D}=diag(d_i)\in\mathbb{R}^{\mathbf{N}\times\mathbf{N}}$ with $d_i=\sum_{i\in\mathcal{N}}a_{ij}$ and the Laplacian matrix as $L=\mathcal{D}-\mathcal{A}$. A digraph is strongly connected, i.e., there is a directed path between any pair of distinct nodes. Moreover, a digraph is called to have a spanning tree, if there is a node v_i (called the root node), such that there is a directed path from the root node to every node in the graph.

Lemma 1. *[2]Zero is an eigenvalue of L with $\mathbf{1}_N$ as a right eigenvector and all nonzero eigenvalues have positive real parts. Furthermore, zero is a simple eigenvalue of L if and only if $\mathcal{G}$ has a directed spanning tree.*

Lemma 2. *[12]Suppose L is a nonsingular M-matrix, then there exists a diagonal matrix Γ such that $\Gamma L+L^{\mathrm{T}}\Gamma=\Phi>0$. One such Γ is given by* diag$(\gamma_1,\gamma_2,\dots,\gamma_N)$, *where $\gamma=[\gamma_1,\gamma_2,\dots,\gamma_N]^{\mathrm{T}}=(L^{\mathrm{T}})^{-1}\mathbf{1}_N$.*

2.2. Node dynamics and communication networks

In this paper, we consider a multi-agent system of a leader node and N follower nodes. The leader subsystem, labeled as $k=0$, is described by

$$\dot{x}_0(t)=Ax_0(t)+Bv_0(t), \tag{1}$$

where $x_0(t)\in\mathbb{R}^n$ represents the state of the leader, $v_0(t)\in\mathbb{R}^m$ represents a smooth bounded reference input. Furthermore, the dynamics of the k-th follower are described by

$$\dot{x}_k(t)=Ax_k(t)+Bu_k(t)+D\omega_k(t),\quad k=1,2,\dots,N, \tag{2}$$

where $x_k(t)\in\mathbb{R}^n$, $u_k(t)\in\mathbb{R}^m$ and $\omega_k(t)\in\mathbb{L}_2^l[0,+\infty)$ are the state, control input, and exogenous disturbance of the k-th follower node, respectively. Suppose A,B and D are constant matrices, and (A,B) is stabilizable.

Assumption 1. There exists a unknown positive constant v^*, such that

$$\|v_0(t)\|\le v^*.$$

Remark 1. As discussed in [18,19,5,20], not all follower nodes in the team can directly observe exact reference trajectory $x_0(t)$, which is generated by leader agent. Moreover, in practical scenario, nonzero control input on the leader contributes to achieve