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Exponential synchronization of discrete-time mixed delay neural networks with actuator constraints and stochastic missing data ☆

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ABSTRACT

This paper investigates the problem of exponential synchronization of discrete-time neural networks with mixed time delays, actuator saturation and failures. Meanwhile, the unreliable communication links are considered between the neural networks, and such unreliable links are modeled as stochastic missing data satisfying Bernoulli distributions. In order to show the relationships between actuator constraints, unreliable communication link and mixed delay neural networks, by using Lyapunov functional approach, a missing data probability dependent exponential synchronization criterion is given. Then, based on such criterion, a reliable controller is designed to ensure that the neural networks are exponentially synchronized in the mean square. Finally, a numerical example is provided to illustrate the effectiveness of the proposed approach.

1. Introduction

During the past few decades, neural networks have been extensively investigated due to the fact that the neural networks have wide application in a variety of fields, such as signal processing, static image processing, pattern recognition, combinatorial optimization and associative memory, see [1–5,10,22] and the references therein. Recently, the time-varying delays that exist between neurons have paid attentions because they are often the sources to cause poor performance and instability of neural networks [6,26]. With further development, another kind of time-delay also attracted considerable interest, namely distributed delay [11,21], and several interesting research results for systems with mixed time delays have been provided, in [7], for neutral system with mixed time-delays and sector-bounded nonlinearity, the stability problem was proposed and the research for stochastic neural system with Markovian jump parameters and mixed delays was given in [8,12]. A sufficient condition has been derived in [18] to ensure the exponential stability in mean square for stochastic neural networks.

On the other hand, the research on synchronization problem for delayed neural networks has been shown to have an important impact on the fundamental science [9,25], such as chemistry, nonlinear oscillation synchronization, secret communication and cryptography and so on [20]. For instance, the problem of adaptive synchronization was investigated in [23] for stochastic neural networks by using the M-matrix approach. In [24], the synchronization problem was investigated for a new class of continuous-time neural networks, and in such systems, all discrete, distributed and the neural delays are mode dependent. However, in the presence of actuator constrains and stochastic data missing, the delay-dependent synchronization problem for discrete-time mixed-delay neural networks has not been fully investigated. The actuator constraints, such as actuator faults and actuator saturation, often appear in a variety of practice, and the synchronization problem is much more complicated. Therefore, the main purpose of this paper is to deal with the synchronization problem of mixed-delay neural networks with actuator constraints and stochastic data missing.

In this article, we will investigate the synchronization problem of discrete-time mixed delay neural networks with actuator faults, actuator saturation and stochastic missing data. The missing data is modeled as a stochastic process satisfying Bernounlli distribution. Considering the characteristic of mixed delay, missing data and actuator constraints, a series of Lyapunov functions are constructed, then, a sufficient condition is given to design a feedback

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controller to ensure exponential synchronization in mean square for neural networks . Finally, a numerical example is shown to illustrate the effectiveness of the proposed method. The main contributions of this paper can be summarized as follows: (1) the error dynamic system is modeled to show the relationships between actuator failures, actuator saturation, mixed time delay and stochastic missing data. (2) A missing data probability dependent exponential synchronization criterion is given for the delayed system subject to actuator constrains and stochastic missing data. (3) The obtained results are extended to uncertain neural networks with actuator constrains and missing data.

Notation: Throughout the paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The notation $X\geq Y$ (or $X>Y$) means that X and Y are symmetric matrices and $X-Y$ is positive semi-definite (or positive definite). The superscript "T" denotes the matrix transposition. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$.

2. Model formulation and preliminaries

Consider the following discrete-time neural network with mixed delays, and it is denoted as the master system, such system can be described as

$$Master:\begin{cases} x(k+1)=Ax(k)+B_0g(x(k))+B_1g(x(k-\tau(k))) \\ \qquad\qquad +C\sum_{m=1}^{+\infty}\mu_m\alpha(x(k-m)) \\ x(k)=\phi_1(k),\quad k=-\tau_M,-\tau_M+1,\dots,-\tau_m \end{cases} \tag{1}$$

where $x(k)=\left[x_1(k),x_2(k),\dots,x_n(k)\right]\in\mathbb{R}^n$ is the neural state vector, $x_i(k)$, $i=1,2,\dots,n$, means the state of the ith neuron at time k. $A=diag\{a_1,a_2,\dots,a_n\}$ is the state feedback coefficient matrix and satisfying $\|a_i\|<1$. $B_1,B_2\in\mathbb{R}^n$ are connection weight matrices and $D\in\mathbb{R}^n$ is the system coefficient matrix, respectively. $g(x(k))$ is the neuron activation function and $\alpha(x(k))$ is the nonlinear function, $g(x(k))=\left[g_1(x_1(k)),g_2(x_2(k)),\dots,g_n(x_n(k))\right]$, $\alpha(x(k))=\left[\alpha_1(x_1(k)),\alpha_2(x_2(k)),\dots,\alpha_n(x_n(k))\right]$, $\phi_1(k)$ is the initial state of the master system. Moreover, $\tau(k)$ denotes the discrete time-varying delay, and satisfying $\tau_m\leq\tau(k)\leq\tau_M$, where τ_m and τ_M are the maximum and the minimum allowed delay constant bound, respectively. μ_m is a kind of nonnegative constant and the convergent conditions are satisfied as [11]:

$$\sum_{m=1}^{+\infty}\mu_m<+\infty,\qquad \sum_{m=1}^{+\infty}m\mu_m<+\infty \tag{2}$$

Throughout the whole paper, we make the following assumptions:

Assumption 1. Given any $x,y\in\mathbb{R}(x\neq y)$, $i\in\{1,2,\dots,n\}$, the activation function $g(x(k))$ and nonlinear function $\alpha(x(k))$ is continuous and bounded, and there exist constants g_i^-, g_i^+, α_i^- and α_i^+ such that

$$g_i^-\leq\frac{g_i(y)-g_i(x)}{y-x}\leq g_i^+,\qquad \alpha_i^-\leq\frac{\alpha_i(y)-\alpha_i(x)}{y-x}\leq\alpha_i^+$$

By considering the master system in (1), the corresponding slave system can be described as the following equation:

$$Slave:\begin{cases} y(k+1)=Ay(k)+B_0g(y(k))+B_1g(y(k-\tau(k))) \\ \qquad\qquad +C\sum_{m=1}^{+\infty}\mu_m\alpha(y(k-m))+\sigma(u^F(k)) \\ y(k)=\phi_2(k),\quad k=-\tau_M,-\tau_M+1,\dots,-\tau_m \end{cases} \tag{3}$$

where A, B_0, B_1 and C are matrices illustrated in (1), $\sigma(u^F(k))$ is the control input under the actuator constraints. Generally speaking, $\sigma(\cdot)$ denotes the actuator saturation, and described as $\sigma_i(r_i)=sign(r_i)\min\{r_{i,max},\|r_i\|\}$, where $r_{i,max}$ denotes the ith element of the vector r_{max}, the saturation level. As [14], if there exist diagonal matrices R_1 and R_2, such that $0\leq R_1<I\leq R_2$, then the saturation function $\sigma(u^F(k))$ can be rewritten as:

$$\sigma(u^F(k))=R_1u^F(k)+\Psi(u^F(k)) \tag{4}$$

Eq. (4) can be divided into two parts, one is a linear part $(R_1u^F(k))$, and another one is a nonlinear function $(\Psi(u^F(k)))$, assume that such nonlinear function satisfies the sector bounded condition, as

$$\Psi^T(u^F(k))(\Psi(u^F(k)-Ru^F(k)))\leq 0,\qquad R=R_2-R_1 \tag{5}$$

In Eq. (5), $u^F(k)\in\mathbb{R}^n$ is the control input subjected to actuator failures. As references [15,16], the controller $u^F(k)$ can be designed as $u^F(k)=Mu(k)$, where $u(k)$ is the controller to be designed in such research and M is the actuator failure matrix and satisfying:

$$M=diag\{m_1,m_2,\dots,m_n\},\qquad 0\leq m_i^{min}\leq m_i\leq m_i^{max}\leq 1,\\ i=1,2,\dots,n \tag{6}$$

If $m_i=1$, this denotes the ith actuator is running without failure, and $m_i=0$ means the ith actuator is outage. Meanwhile, $0<m_i<1$ represents that the ith actuator has partial failure. Thus, the model of the slave system can be reconstructed as:

$$Slave:\begin{cases} y(k+1)=Ay(k)+B_0g(y(k))+B_1g(y(k-\tau(k))) \\ \qquad\qquad +C\sum_{m=1}^{+\infty}\mu_m\alpha(y(k-m))+R_1Mu(k) \\ \qquad\qquad +\Psi(u^F(k)) \\ y(k)=\phi_2(k),\quad k=-\tau_M,-\tau_M+1,\dots,-\tau_m \end{cases} \tag{7}$$

with the sector-bounded constraints

$$\Psi^T(u^F(k))(\Psi(u^F(k)-RMu(k)))\leq 0,\qquad R=R_2-R_1 \tag{8}$$

By defining the error signal as $e(k)=y(k)-x(k)$, the error dynamic system can be obtained as follows:

$$\begin{cases} e(k+1)=Ae(k)+B_0f(e(k))+B_1f(e(k-\tau(k))) \\ \qquad\qquad +C\sum_{m=1}^{+\infty}\mu_m\beta(e(k-m))+R_1Mu(k) \\ \qquad\qquad +\Psi(u^F(k)) \\ e(k)=\phi_2(k)-\phi_1(k),\quad k=-\tau_M,-\tau_M+1,\dots,-\tau_m \end{cases} \tag{9}$$

where $f(e(k))=g(y(k))-g(x(k))$, $\beta(e(k))=\alpha(y(k))-\alpha(x(k))$. Based on Assumption 1, it can be found that

$$g_i^-\leq\frac{f_i(\nu)}{\nu}\leq g_i^+,\qquad \alpha_i^-\leq\frac{\beta_i(\nu)}{\nu}\leq\alpha_i^+ \tag{10}$$

where $\nu\in\mathbb{R}$, and $\nu\neq 0$. In practical, the unreliable communication links exist in the considered system, thus, random data loss may exist, and we use a stochastic variable $\theta(k)$ to describe the data loss phenomena at time k, thus the controller is designed as

$$u(k)=\theta(k)Ke(k) \tag{11}$$

where $K\in\mathbb{R}^{n\times n}$ is the designed controller gain matrix, and assume that $\theta(k)$ satisfies the Bernounlli distribution as [13], $\theta(k)=1$ when the controller data is received, whereas $\theta(k)=0$ denotes the data is missing. However, in Eq. (9), M is not known in advance, thus, we have defined $M=M_0(I+G)$, $\|G\|\leq H\leq I$ as [15], where