Information Sciences 369 (2016) 419–440

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Evolutionary algorithms and elliptical copulas applied to continuous optimization problems

Harold Dias de Mello Junior[a,*], Luis Martí[b], André V. Abs da Cruz[c], Marley M.B. Rebuzzi Vellasco[d]

[a] *Electrical Engineering Department, Rio de Janeiro State University, Rio de Janeiro, Brazil*
[b] *Institute of Computing, Fluminense Federal University, Rio de Janeiro, Brazil*
[c] *Computer University Unit, State University of West Side, Rio de Janeiro, Brazil*
[d] *Electrical Engineering Department, Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro, Brazil*

ARTICLE INFO

Article history:
Received 14 October 2015
Revised 28 June 2016
Accepted 2 July 2016
Available online 4 July 2016

Keywords:
Continuous numeric optimization
Copulas
Estimation of distribution algorithms
Evolutionary computation

ABSTRACT

Estimation of Distribution Algorithms (EDAs) constitutes a class of evolutionary algorithms that can extract and exploit knowledge acquired throughout the optimization process. The most critical step in the EDAs is the estimation of the joint probability distribution associated to the variables from the most promising solutions determined by the evaluation function. Recently, a new approach to EDAs has been developed, based on copula theory, to improve the estimation of the joint probability distribution function. However, most copula-based EDAs still present two major drawbacks: focus on copulas with constant parameters, and premature convergence. This paper presents a new copula-based estimation of distribution algorithm for numerical optimization problems, named EDA based on Multivariate Elliptical Copulas (EDA-MEC). This model uses multivariate copulas to estimate the probability distribution for generating a population of individuals. The EDA-MEC differs from other copula-based EDAs in several aspects: the copula parameter is dynamically estimated, using dependence measures; it uses a variation of the learned probability distribution to generate individuals that help to avoid premature convergence; and uses a heuristic to reinitialize the population as an additional technique to preserve the diversity of solutions. The paper shows, by means of a set of parametric tests, that this approach improves the overall performance of the optimization process when compared with other copula-based EDAs and with other efficient heuristics such as the Covariance Matrix Adaptation Evolution Strategy (CMA-ES).

1. Introduction

Optimization concepts and tools have been widely used in quantitative methods to address extensive classes of decision support problems, from various fields of knowledge, especially in engineering. In the absence of computationally treatable deterministic solution proposals, stochastic algorithms become a viable alternative for their high chances of identifying a good solution and even of reaching the global optimum in a reasonably short time. Among the many stochastic global

* Corresponding author.
E-mail address: harold.dias@gmail.com (H.D. de Mello Junior).

http://dx.doi.org/10.1016/j.ins.2016.07.006
0020-0255/

optimization algorithms [57,62], some are nature-inspired metaheuristics. The most well-known ones are swarm intelligence and evolutionary algorithms.

Swarm intelligence algorithms take inspiration from the collective behavior of social insects and other animal societies [5]. These algorithms assume a population of simple individuals/agents capable of jointly producing a self-organized behavior for their survival. The most successful swarm intelligence inspired optimization algorithms are ant colony (ACO) [13] for discrete problems, and particle swarm optimization (PSO) [29] for continuous problems, including real-world tasks.

Evolutionary algorithms, such as genetic algorithms [38], are also population-based metaheuristics of great research interest because of their promising results in different applications, such as telecommunications [27]. These metaheuristics use principles of Darwin's theory of natural selection: at each generation or iteration of the algorithm, a competitive selection occurs to choose the best solutions; these are modified by operators to generate new solutions, repeating this cycle until a given stop criterion, defined by the user, is reached.

In the same group of population-based metaheuristics are the Estimation of Distribution Algorithms (EDAs). EDAs [25,42] exploit promising regions of the search space by sampling the probabilistic model, which is estimated previously and periodically from a set of candidate solutions selected by the algorithm according to their fitness values. This learning mechanism and the statistical sampling allow EDAs to detect explicitly the dependence relationship between decision variables, transferring it to the new solutions of the next generation, and to have better performance than other evolutionary algorithms since the optimization uses, in general, a smaller number of objective function evaluations [33].

The ability to solve large and complex problems with superior performance than other global optimization techniques [25] has been a great motivating factor for the development of new approaches to this type of algorithm, all of them seeking to add estimation methods with the best trade-off between accuracy on the one hand and computational cost of learning and sampling of the probabilistic model on the other. In the recent years, different techniques have been proposed to improve its estimation [25,31], including but not limited to models based on copula theory. Copulas, according to Sklar's theorem [53], are used to build a joint probability distribution relating the dependence structure (established by a copula) to its marginal univariate distributions, instead of assuming unrealistic Gaussian distributions as a proxy for the joint probability distribution. This technique reduces the computational effort for model learning and gives greater flexibility in modeling multivariate data, generating important contributions in various applications, such as financial risk management [11]. In the context of evolutionary computing, the use of copulas in EDAs is more recent and therefore the subject of much research interest. Copula-based EDAs [20] start by calculating the marginal distribution of each variable separately. They then pick a particular copula to construct the joint distribution. Given the marginal distributions and a copula, it is then possible to generate new candidate solutions. Despite the good results achieved in tests with some numerical optimization benchmarks, the current copula-based EDAs present two major limitations. First, the copula parameters are assumed to be constant, which reduces the algorithm's performance. The second limitation concerns premature convergence to a local optimum. Although the capacity of the class of probability distribution used in the EDAs has grown significantly, real-valued copula-based EDAs tend to converge prematurely on problems such as the Rosenbrock problem [9]. Moreover, aspects of modeling copulas in EDAs need still to be investigated, including the limitations of building multivariate copulas, efficiency in the relationship between the type of copula and the marginal distributions, and mechanisms to maintain the population diversity to make it a competitive tool for global optimization.

This paper presents a new approach to copula-based EDAs that is more efficient than other similar algorithms and very promising when compared with one of the best evolutionary algorithms for numerical optimization problems, the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [21,22]. The proposed approach, named EDA based on Multivariate Elliptical Copulas (EDA-MEC), is based on elliptical copulas, such as the multivariate Gaussian and Student's t copulas. They are widely used in empirical applications due to their tractable properties similar to those of multivariate normal distributions. EDA-MEC considers the construction of probabilistic models with a better trade-off between accuracy and computational effort and differs from the others in several respects: the copula parameters are updated dynamically in every algorithm generation, generating more reliable search distributions; the approach makes use of variations of the estimated probabilistic model to insert diversity into the population, avoiding premature convergence; and it uses an adaptive heuristic to reinitialize the population throughout an elitist evolution as an additional mechanism for diversity.

The structure of the paper is the following: Section 2 presents the mechanisms and characteristics of the main evolutionary algorithms, including EDAs. Section 3 provides a brief introduction to copulas and copula-based EDAs. Section 4 provides details of the proposed model, the Estimation of Distribution Algorithm based on Multivariate Elliptical Copulas (EDA-MEC). Section 5 presents the experimental setting to solve a test suite for numerical optimization, compares the EDA-MEC performance to other models, and discusses the results. Finally, Section 6 articulates some general conclusions.

2. Fundamentals of evolutionary algorithms

Evolutionary computation is a subfield of computational intelligence, covering various techniques inspired by biological evolution mechanisms; these are considered stochastic methods of global optimization. Most of these methods use population-based metaheuristics [39,57]. Algorithms such as evolutionary algorithms, collective intelligence algorithms (swarm computing) and memetic algorithms belong to this class of metaheuristics.

The most significant advantage of evolutionary computing is the robustness and flexibility of its algorithms based on generic and adaptable procedures, which allow solving complex problems for which traditional methods are ineffective.