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Cooperative coevolutionary differential evolution with improved augmented Lagrangian to solve constrained optimisation problems

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A B S T R A C T

In constrained optimisation, the augmented Lagrangian method is considered as one of the most effective and efficient methods. This paper studies the behaviour of augmented Lagrangian function (ALF) in the solution space and then proposes an improved augmented Lagrangian method. We have shown that our proposed method can overcome some of the drawbacks of the conventional augmented Lagrangian method. With the improved augmented Lagrangian approach, this paper then proposes a cooperative coevolutionary differential evolution algorithm for solving constrained optimisation problems. The proposed algorithm is evaluated on a set of 24 well-known benchmark functions and five practical engineering problems. Experimental results demonstrate that the proposed algorithm outperforms the state-of-the-art algorithms with respect to solution quality as well as efficiency.

1. Introduction

Most of the science and engineering optimisation problems in real world are highly constrained. These constrained optimisation problems present serious challenges to existing optimisation methods. Developing effective constraint handling techniques is critical in addressing these challenges. Constraint handling techniques can be categorised into four groups [26]: maintaining feasibility of solutions, penalty functions, distinguishing between feasible and infeasible solutions and hybrid methods. Other categorisations are also possible [8]. Generally speaking, each of these techniques has some advantages and disadvantages.

Evolutionary algorithms (EAs) have been applied to various optimisation problems which classical optimisation algorithms cannot be directly applied to or do not provide promising results [40]. One of the most common constraint handling techniques with EA is the penalty function approach, as presented by Courant et al. [10]. The penalty function approach converts a constrained optimisation problem into a sequence of unconstrained problems by adding a penalty term to the original objective function to penalise infeasible solutions [28]. However penalty functions are often not differentiable and this is the main drawback of using this approach. Another popular approach is the Lagrangian multiplier method, which is based on Kuhn-Tucker conditions and can be used to convert a constrained optimisation problem into an unconstrained

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one. However, this approach assumes the problem to be convex. In order to handle non-convex problems, the augmented Lagrangian method is introduced in [38], to convexify the objective function by adding quadratic penalty terms [44].

Many studies have been carried out during the last decade to solve constrained optimisation problems using the augmented Lagrangian approach. Adeli and Cheng [1] proposed a hybrid genetic algorithm (GA) to solve structural optimisation using ALF. Sarma and Adeli [42] presented a fuzzy augmented Lagrangian method using GA to optimise steel structures. Rocha et al. [37] proposed a stochastic population based algorithm using the augmented Lagrangian method. An artificial fish swarm algorithm based hyperbolic augmented Lagrangian method is used to solve constrained optimisation problems in [9]. An ant colony optimisation (ACO) algorithm with augmented Lagrangian method is presented in [21] to solve continuous global optimisation problems. Mallipeddi and Suganthan [22] presented an ensemble of four different constraint handling methods to solve constrained optimisation problems.

Dealing with complex combinatorial solution spaces as well as problems with high number of constraints is a challenging task. Some attempts have been made to hybridise the EAs with local search algorithms to cope with this challenge in an efficient way [40] e.g., a hybrid particle swarm optimisation(PSO) with GA [14]. Another approach to deal with the aforementioned challenge is using a coevolutionary algorithm. Tahk and Sun [44] presented a coevolutionary algorithm using zero-sum game to coevolve the decision variables and Lagrangian multipliers. A coevolutionary GA has been also used to solve constrained optimisation problems [3]. Krohling et al. [19] used a coevolutionary PSO augmented Lagrangian function to deal with constraints. They proposed a Gaussian probability distribution for the acceleration coefficient in PSO. Nema et al. [29] presented a hybrid coevolutionary algorithm with the min-max approach to solve constrained optimisation problems. They also used an augmented Lagrangian method to handle constraints.

Although ALF is an efficient method to deal with constraints, it changes the fitness values dramatically for solutions lying far from the boundaries of the feasible space. In this paper, we propose an improved augmented Lagrangian function (iALF) to handle this issue in a more effective manner. Based on iALF, we also propose an efficient CCiALF method for solving constrained optimisation problems. The proposed algorithm produces higher quality solutions using fewer number of function evaluations (NFE). To demonstrate the capability of CCiALF algorithm, two sets of benchmarks are used in our study and the results are compared with that of the state-of-the-art algorithms.

The rest of the paper is structured as follows: First some background on ALF is described in Section 2, and then our improved ALF (iALF) is presented in section 3. The proposed CCiALF method is introduced in Section 4 and experimental results are presented in Section 5. Finally, Section 6 provides conclusion and future research directions.

2. Augmented Lagrangian function

The general constrained optimisation problem can be described as follows:

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}), \qquad \boldsymbol{x} \in R^p \tag{1}$$

$$g_i(\boldsymbol{x}) \geq 0, \qquad i = 1, \ldots, m \tag{2}$$

$$h_j(\boldsymbol{x}) = 0, \qquad j = 1, \ldots, n \tag{3}$$

$$lb_k \leq x_k \leq ub_k, \quad k = 1, \ldots, p \tag{4}$$

where Eq. (1) represents the objective function, Eqs. (2) and (3) are inequality and equality constraints, respectively. Eq. (4) represents lower and upper bounds on decision variables $\boldsymbol{x}$. In [16] and [31], only the equality constraints are considered and the above problem is transformed into an unconstrained one by adding quadratic penalty terms and dual values to the objective function. Rockafellar [39] utilised the idea and modified it for inequality constraints. The augmented Lagrangian (also called as penalty Lagrangian by Rockafellar [38]) replacing the quadratic penalty term by θ function is presented as follows [13]:

$$F(\boldsymbol{x}, \boldsymbol{\mu}, \boldsymbol{\tau}) = f(\boldsymbol{x}) + R\sum_{j=1}^{m}[(\theta(g_j(\boldsymbol{x}) + \mu_j))^2 - (\mu_j)^2] + R\sum_{k=1}^{n}[(h_k(\boldsymbol{x}) + \tau_k)^2 - (\tau_k)^2] \tag{5}$$

$$\theta(G) = min\{0, G\} \tag{6}$$

where R is a positive penalty parameter, $\boldsymbol{\mu}$ is a $1 \times m$ multiplier, $\boldsymbol{\tau}$ is a $1 \times n$ multiplier for inequality and equality constraints respectively, and G can be any function or value. The θ function checks if the inner expression (e.g. G) is greater than zero or not. If $G \geq 0$ then $\theta(G) = 0$, otherwise $\theta(G) = G$.

Deb and Srivastava [13] evaluated the classical ALF using benchmark functions and it is shown that the classical ALF has limited success, e.g., the solutions are still far from the known optima. Fig. 1 shows an example of a 2-dimensional problem where ALF divides the search space into four different regions: the inner feasible region, ie., the inner feasible area far from the boundary (region 1), the feasible area close to the boundary (region 2), the feasible area on the boundary(region 3) and finally the infeasible area (region 4).