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LMI-based control synthesis of constrained Takagi–Sugeno fuzzy systems subject to $\mathcal{L}_2$ or $\mathcal{L}_\infty$ disturbances

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ABSTRACT

This paper is devoted to the development of a new saturated non-parallel distributed compensation control law for disturbed Takagi–Sugeno fuzzy systems subject to both control input and state constraints. In order to cover a large range of real-world applications, both $\mathcal{L}_2$ and $\mathcal{L}_\infty$ disturbances are considered which result in two different control design procedures. A parameter-dependent version of the generalized sector condition is effectively exploited in a fuzzy Lyapunov control framework to handle the control input saturation. Moreover, the proposed control method is based on the concept of robust invariant set which is able to provide an *explicit* characterization of the estimated domain of attraction of the closed-loop system. Different optimization algorithms are also proposed to deal with the trade-off between different closed-loop requirements in a local control context. The design conditions are expressed in terms of linear matrix inequalities which can be solved efficiently with available solvers. The numerical examples illustrate how the proposed methodology leads to less conservative results as well as less computational complexity when compared to very recent works in the literature.

1. Introduction

Nowadays, the stability analysis and control design based on Takagi–Sugeno (T–S) fuzzy models [1] have become the most popular research platform in fuzzy model-based control [2]. Indeed, over the past two decades tremendous investigations have been devoted to the study of T–S control systems [2–8]. This fact is due to many outstanding features of T–S fuzzy models for control purposes [3]. First, they can be used to approximate any smooth nonlinear system with any given accuracy. In particular, the sector nonlinearity approach provides an exact T–S representation of a given nonlinear model in a compact set of the state variables. Second, thanks to its polytopic structure with linear systems in consequent parts, T–S representation allows for some possible extensions of linear control techniques to nonlinear systems.

The direct Lyapunov method has been efficiently exploited to study the stability and control synthesis of T–S fuzzy systems [2,6–10]. The derived conditions are expressed in terms of linear matrix inequalities (LMIs) [11] such that they are efficiently solvable with available numerical solvers. It is noteworthy that depending on the choice of the Lyapunov function, the derived conditions have different degrees of conservativeness. The following three types of Lyapunov functions have been mainly investigated in the T–S fuzzy control framework, namely quadratic, piecewise and parameter-dependent Lyapunov functions [2]. Despite the low-complexity of the derived conditions [3], quadratic Lyapunov functions lead generally to conservative results [12]. Piecewise Lyapunov functions [13] could be applied to overcome this major drawback. However, this type of Lyapunov functions requires the membership functions to induce a polyhedral partition of the state space. This fact is not compatible with T–S fuzzy models obtained from original nonlinear systems by using the sector nonlinearity approach [14]. As a consequent, piecewise Lyapunov functions can only be used to deal with nonlinear systems in the sense of approximation. Recent LMI alternative methodology to stability conditions considering piecewise Lyapunov functions is presented in [15]. However, control design conditions based on piecewise Lyapunov functions are in general expressed in terms of bilinear matrix inequalities [16,17] which are hardly tractable with available numerical solvers. The effectiveness of parameter-dependent Lyapunov functions for stability analysis and control design has been demonstrated in [9,18–21]. This type of Lyapunov functions seems to be the best alternative to solve all drawbacks of both previous ones, especially for discrete-time T–S fuzzy systems [8,12,19,22].

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Physical constraints such as control input saturation and system state constraints are ubiquitous in real-world applications due to safety and/or economic reasons. The presence of input saturation seriously degrades the closed-loop performance, in the extreme case, the stability may be lost [23]. However, this practical control issue has not been completely addressed for T–S fuzzy control systems [10,24–26]. Some notable works can be cited as follows. In [27–29], a norm-bounded approach was used to deal with the actuator saturation. The resulting non-saturated controllers are generally very conservative and often lead to poor closed-loop performance [23,30]. Descriptor representation approach [17] has been recently employed to deal with continuous-time input-saturated T–S fuzzy systems in [31]. It should be noted that this result is only applied to a restrictive class of T–S fuzzy systems with all linear subsystems being open-loop stable. The saturation function was represented in polytopic form to deal with input nonlinearity of continuous-time T–S systems [30,32,33], then extended to time-delay T–S systems [34] and a class of switching T–S systems [5,25]. However, based on quadratic Lyapunov functions these results could be conservative. It should be stressed that state constraints were not considered in most of these works (except for [25]). Such type of constraints appears naturally when the sector nonlinearity approach is used to obtain T–S representation of nonlinear systems [35]. Explicit consideration of these limitations allows to prevent destabilizing initial conditions of the closed-loop systems [26]. Especially, this becomes crucial when disturbance signals are actively involved in the systems [10]. Recently, interesting non-quadratic boundedness approach has been also proposed in [24] to deal with T–S fuzzy systems subject to both control input and state constraints. Notice that the results proposed in [24] require several line searches to solve the design conditions which are costly in terms of computation. Moreover, slack decision variables have been intensively introduced in [24] to reduce the conservatism of the results. Therefore, the resulting design conditions are of high complexity and not suitable for high dimensional T–S systems or T–S systems with important number of subsystems. These facts will be clearly shown in Section 5 by means of a numerical example. It is also important to highlight that the method in [24] cannot deal with the case where T–S systems are subject to $\mathcal{L}_2$ disturbances.

Motivated by the above control issues, this paper is devoted to the development of a new input-saturated control law for disturbed T–S fuzzy systems subject to both control input and state constraints. Differently from [24], the proposed method is based on the concept of robust invariant set [36]. The main contributions of the new method can be summarized as follows:

- A parameter-dependent version of the generalized sector condition has been effectively exploited in the framework of fuzzy Lyapunov function based control design to handle the actuator saturation. This fact leads to less conservative design conditions with low computational complexity compared to existing works dealing with the same class of problem.
- The new method can provide an *explicit characterization* of the estimated domain of attraction of the closed-loop system which is not the case of [24].
- The proposed results can be applied to T–S systems subject to $\mathcal{L}_\infty$ or $\mathcal{L}_2$ disturbances. Numerical examples illustrate that the proposed methodology can be applied to a large class of nonlinear systems and suitable for real-world-applications.

The paper is organized as follows. Section 2 formulates the control problem and some useful preliminaries are also presented. In Section 3, we develop new non-quadratic design conditions for two different cases corresponding to two types of disturbances affecting the constrained T–S systems. Optimization algorithms for different control design purposes are presented in Section 4. The interests of the proposed method are clearly demonstrated by means of examples in Section 5. Finally, Section 6 provides some concluding remarks.

Notation. For an integer number r, Ω_r denotes the set $\{1,2,\dots,r\}$. I denotes the identity matrix of appropriate dimension. For any square matrix X, $\mathrm{He}(X)=X+X^\top$. $X>0$ means that the matrix X is positive definite. The ith element of a vector u is denoted $u_{(i)}$ and $X_{(i)}$ denotes the ith row of matrix X. $(*)$ stands for matrix blocks that can be deduced by symmetry. For a positive definite function $\mathbb{V}(x)$ defined on $\mathbb{R}^{n_x}$, we denote $\mathcal{E}_{\mathbb{V}\rho}=\{x\in\mathbb{R}^{n_x}:\mathbb{V}(x)\le\rho\}$ and $\mathcal{E}_{\mathbb{V}}\equiv\mathcal{E}_{\mathbb{V},1}$. The scalar functions η_i, $i\in\Omega_r$, are said to verify the convex sum property on a set $\mathcal{D}$, if $\eta_i(\theta)\ge 0$ and $\sum_{i=1}^r\eta_i(\theta)=1$ for $\forall\theta\in\mathcal{D}$. For such functions and for matrices Y_i and Z_{ij} of appropriate dimensions, we denote

$$Y_\theta=\sum_{i=1}^r\eta_i(\theta(t))Y_i,\quad Z_{\theta\theta}=\sum_{i=1}^r\sum_{j=1}^r\eta_i(\theta(t))\eta_j(\theta(t))Z_{ij}$$
$$Y_\theta^{-1}=\left(\sum_{i=1}^r\eta_i(\theta(t))Y_i\right)^{-1},\quad Y_{\theta+}=\sum_{i=1}^r\eta_i(\theta(t+1))Y_i \tag{1}$$

Throughout this paper, the time argument will be dropped when convenient.

2. Problem formulation and preliminaries

2.1. Problem formulation

In this paper, the fuzzy model proposed in [1] is used to approximate and/or represent a given nonlinear system. This type of model is described by fuzzy IF–THEN rules which represent local linear input–output relations of a nonlinear system. The ith rules of the discrete-time T–S fuzzy system subject to control input saturation can be represented in the following form

Model rule i:
IF θ_1 is M_{i1} and … and θ_p is M_{ip}

$$\text{THEN}\quad\begin{cases}x(t+1)=A_ix(t)+B_i^u\mathrm{sat}(u(t))+B_i^w w(t)\\ z(t)=C_ix(t)\end{cases} \tag{2}$$

where $\mathrm{sat}(u_{(l)})=\mathrm{sign}(u_{(l)})\min(|u_{(l)}|,u_{\max(l)})$, $l\in\Omega_{n_u}$, and M_{ij}, $i\in\Omega_r$, $j\in\Omega_p$, is the fuzzy set and r is the number of model rules; $x\in\mathbb{R}^{n_x}$ is the state, $u\in\mathbb{R}^{n_u}$ is the control input, $w\in\mathbb{R}^{n_w}$ is the system disturbance, $z\in\mathbb{R}^{n_z}$ is the performance output, and $\theta=[\theta_1,\dots,\theta_p]^\top\in\mathbb{R}^p$ is the vector of premise variables. The real matrices A_i, $B_i{}^u$, $B_i{}^w$, C_i, $i\in\Omega_r$, are constant and of adequate dimensions. Then, the T–S fuzzy system is defined as follows:

$$\begin{cases}x(t+1)=\sum_{i=1}^r\eta_i(\theta)\left(A_ix(t)+B_i^u\mathrm{sat}(u(t))+B_i^w w(t)\right)\\ z(t)=\sum_{i=1}^r\eta_i(\theta)C_ix(t)\end{cases} \tag{3}$$

where the normalized membership functions $\eta_i(\theta)$, $i\in\Omega_r$, are defined as

$$\eta_i(\theta)=\frac{\lambda_i(\theta)}{\sum_{j=1}^r\lambda_j(\theta)},\quad \lambda_j(\theta)=\prod_{l=1}^p M_{lj}(\theta) \tag{4}$$

In (4), $M_{lj}(\theta)$ denotes the membership function of fuzzy set M_{lj}. It is worth noting that the normalized membership functions $\eta_i(\theta)$, $i\in\Omega_r$, satisfy the convex sum property.

Remark 1. T–S fuzzy system is a class of fuzzy systems where the consequent parts are *functions* of premise variables [1]. These functions can be linear or affine as most of the cases in fuzzy control framework [2,3]. However, T–S fuzzy systems with local