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Group consensus for second-order discrete-time multi-agent systems with time-varying delays under switching topologies ☆

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ABSTRACT

This paper addresses group consensus for multi-agent systems with switching topologies and time-varying delays, where all agents are described by discrete-time second-order dynamics. Based on the relationship between zero/nonzero in-degree graphs and nonnegative matrices group consensus criteria, which is presented in terms of easily checkable graph topology conditions, is obtained for multi-agent systems with time-varying delays under switching topologies. Furthermore, numerical examples are presented to illustrate the obtained results, and the essential of some conditions and assumptions is verified for given switching communication topologies.

1. Introduction

Recently, the consensus problem of multi-agent systems has received an increased interest motivated by the broad range of application involving unmanned air vehicles, formation control of mobile robots, flocking of social insects. Here, consensus refers to matching the states of the systems, or agents, to a common final state which is determined by agreement between the agents. The information flows in the network, described generally by an interconnection/communication graph, plays a central role. This has been demonstrated in existing results on the consensus of multi-agent systems, which include [1–4] to name only a few.

Based on the reference articles, many theoretical results have been achieved on complete consensus problems for multi-agent systems [6–23]. Normally, consensus is defined as a general agreement among all members of a given group or community, each of which exercises some discretion in decision making and in its interactions with other agents. In [6], the authors provided a valid distributed consensus algorithm that overcomes the difficulties caused by unreliable communication channels. Following this line, there have been numerous results reported and several effective approaches proposed over the past few years, including matrix analysis [20], graph theory [18], and the construction of Lyapunov function [11]. In fact, in 2007, Ren and Atkins put forward a typical second-order system in [21]. As the questions about consensus for second-order multi-agent systems were presented, many articles appeared rapidly [22,23].

Note that the above literatures all considered such consensus where the states of all agents converge to the same consensus value. However, a real-world complex network may be composed of interaction smaller subnetworks. The phenomenon of group (cluster) consensus is observed when an ensemble of oscillators splits into several subgroups, called group consensus throughout the paper. Due to the changes of situations or cooperative tasks, the consensus values may be different for agents from different sub-networks [24–38]. Group consensus was introduced in [24] to represent such consensus where the states of all agents in the same sub-network reach the same consistent value while there is no agreement among different sub-networks. Average group consensus problem for multi-agent system with undirected and balanced topologies was investigated in [26]. More results about group consensus problems subsequently were obtained by many outstanding researchers [28,29,33,35–37]. By using algebraic graph theory and Lyapunov approaches, the second-order cluster consensus for multi-agent systems with inter-identical inputs is investigated in [33]. Reference to another typical second-order system, raised by Xie and Wang in [34], an input controller was designed for couple group consensus problem in [35]. And couple-

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group consensus was solved based on Lyapunov method and Hopf bifurcation theory, ultimately, the static group consensus state was achieved. In [38], the couple-group consensus problem for multi-agent networks with fixed and directed communication topology was investigated, and for a given communication topology, a theorem is derived on how to select proper control parameters and sampling period for couple-group consensus to be reached.

Inspired by the group consensus phenomenon in complex networks, this paper aims to further investigate the group consensus of second-order discrete-time multi-agent systems. Quite different from the current references, this paper addresses a second-order discrete-time multi-agent system, few consensus results have been published about it. It is worthy to be pointed out that as opposed to the analysis conditions for which the feasibilities are difficult to be checked, all the results in this paper are presented in terms of easily checkable graph topology conditions.

The rest of this paper is organized as follows. In Section 2, some preliminaries and problem formulation are introduced. The result about group consensus for system with fixed topology is presented in Section 3. The result about group consensus for system with switching topologies and time-varying delays is derived in Section 4. Some simulation results are presented in Section 5 to demonstrate the effectiveness of theoretical results. Section 6 concludes this paper.

Notations: In the remainder of the paper, we use the following notations. I_n denotes the identity matrix with dimension n; $\mathbf{0} = (0, \ldots, 0)^T$, $\mathbf{1} = (1, \ldots, 1)^T$ with an appropriate dimension; the operation $\otimes$ is the Kronecker product. $A_0 = diag(A)$ is a diagonal matrix with all diagonal elements of matrix A. $\| z \|$ denotes the normal of a vector z and $\| A \|$ denotes the matrix norm of A induced by the vector norm $\|\cdot\|$.

2. Preliminaries and problem formulation

In this section, we first present some notations and definitions that are related to graph and matrix theories.

Here, a directed graph $\mathcal{G}$ consists of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and a directed edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, i.e. an edge is an ordered pair of vertices in $\mathcal{V}$ and $\mathcal{A} = [a_{ij}]$, $a_{ij>0}$ is a weighted adjacency matrix. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in \mathcal{E}$ if and only if $a_{ij} > 0$. We say that $\mathcal{G}$ has self-loop if $(v_i, v_i) \in \mathcal{E}$ for all $i \in \mathcal{F}$. The node indexes belong to a finite index set $\mathcal{F} = \{1, 2, \ldots, n\}$. The set of neighbors of node v_i is denoted by $N_i = \{v_j \in \mathcal{V}: (v_i, v_j) \in \mathcal{E}\}$. A directed path in $\mathcal{G}$ from i_1 to i_l is a sequence of ordered edges in the form of $(v_{i_k}, v_{i_{k+1}})$, $k = 1, 2, \ldots, l-1$, where $(v_{i_k}, v_{i_{k+1}}) \in \mathcal{E}$. The graph contains a spanning tree if there exists a vertex v_i such that all the other vertices v_j there is a directed path from v_i to v_j, and v_i is called the root vertex.

In this paper, we define a grouping $\mathcal{C}$ as a disjoint division of the vertex set, namely, a sequence of subsets of $\mathcal{V}$, $\mathcal{C} = \{\mathcal{C}_1, \ldots, \mathcal{C}_K\}$, that satisfies (i) $\bigcup_{p=1}^{K} \mathcal{C}_p = \mathcal{V}$; (ii) $\mathcal{C}_k \cap \mathcal{C}_p = \varnothing$, $k \le p$, $k, p \in \{1, 2, \ldots, K\}$.

Consider a multi-agent system consisting of n autonomous agents. Suppose the dynamics of the ith agent is

$$x_i(t+1) = x_i(t) + v_i(t)T,$$
$$v_i(t+1) = v_i(t) + u_i(t)T, \quad i \in \mathcal{F}, \tag{1}$$

where $x_i(t) \in R$, $v_i(t) \in R$ describe position and velocity of the ith agent, respectively. $u_i(t) \in R$ is the control input, T is the sample period.

To solve the group consensus problem, we introduce the following protocol which uses the local information and the distributed relative state information

$$u_i(t) = -k_0 v_i(t) + \sum_{j=1}^{n} a_{ij}(t)[(x_j(t - \tau_{ij}(t)) - x_i(t)) + k_1(v_j(t - \tau_{ij}(t)) - v_i(t))], \quad i \in \mathcal{C}_p,\ p = 1, 2, \ldots, K, \tag{2}$$

where $k_0 > 0$ denotes the velocity damping gain, $k_1 > 0$. $a_{ij}(t)$ denotes the edge weight chosen from a finite set. Here, $\tau_{ij}(t) \in Z_+$, $\tau_{ij}(t) \le \tau_{max}$, where τ_{max} denotes the maximum communication time delays.

We say that protocol $u_i(t)$ asymptotically solves the group consensus problem, if the states and velocities of agents satisfy

(a_1) $\lim_{t\to\infty}[x_j(t) - x_i(t)] = 0, \quad \forall\, i, j \in \mathcal{C}_p,\ p = 1, 2, \ldots, K;$

(a_2) $\lim_{t\to\infty} v_i(t) = 0, \quad i \in \mathcal{F}.$

In this paper, achieving group consensus criteria for second-order discrete-time multi-agent systems is a tough goal with the impact of velocity and time-varying delays. To facilitate the statement of main results, we first in sequence present the convergent analysis on the system under fixed topology.

3. Group consensus with fixed topology

In the following, we will assume that a fixed protocol denoted as

$$u_i(t) = -k_0 v_i(t) + \sum_{j=1}^{n} a_{ij}[(x_j(t) - x_i(t)) + k_1(v_j(t) - v_i(t))], \quad i \in \mathcal{C}_p,\ p = 1, 2, \ldots, K. \tag{3}$$

To reach the desired group pattern under distributed control law (3), we use model transformation technique and topology graph transformation in the analysis below.

Let $\bar{v}_i(t) = x_i(t) + k_1 v_i(t)$, and then $\lim_{t\to\infty}\bar{v}_i(t) = \lim_{t\to\infty}x_i(t)$. Moreover, (1) is equivalent to

$$x_i(t+1) = x_i(t) + \frac{\bar{v}_i(t) - x_i(t)}{k_1}T,$$
$$\bar{v}_i(t+1) = \bar{v}_i(t) + \frac{1 - k_0 k_1}{k_1}T(\bar{v}_i(t) - x_i(t)) + k_1 T\sum_{j=1}^{n} a_{ij}(\bar{v}_j(t) - \bar{v}_i(t)), \quad i \in \mathcal{C}_p,\ p = 1, 2, \ldots, K. \tag{4}$$

Furthermore, suppose that

$$E = \begin{bmatrix} 1 - \frac{T}{k_1} & \frac{T}{k_1} \\ -\frac{1 - k_0 k_1}{k_1}T & 1 + \frac{1 - k_0 k_1}{k_1}T \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 \\ 0 & k_1 T \end{bmatrix},$$

and $y(t) = [x_1(t), \bar{v}_1(t), x_2(t), \bar{v}_2(t), \ldots, x_n(t), \bar{v}_n(t)]^T$, and then we can obtain

$$y(t+1) = [I_n \otimes E - \mathcal{L} \otimes F]y(t), \tag{5}$$

where $\mathcal{L}$ is the Laplacian matrix of $\mathcal{G}(\mathcal{A})$ and satisfies the following assumption.

Assumption 1. The adjacency matrix $\mathcal{A}$ of graph $\mathcal{G}$ has inter-group common influence [32].

Denote $d_{max} = \max_{1\le i\le n}\{l_{ii}\}$, suppose parameters k_0, k_1 satisfy

$$T d_{max} k_1^2 + (k_0 T - 1)k_1 - T < 0, \quad k_1 > T \text{ and } k_1 > \frac{1}{k_0}. \tag{6}$$