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Further results on absolute stability of Lur'e systems with a time-varying delay[☆]

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ABSTRACT

In this paper, we focus on the problem of absolute stability of Lur'e systems with time-varying delay and sector-bounded nonlinearity. An improved free-matrix-based inequality (FMBI) is derived. By using this inequality and the convex combination technique, some new delay-dependent absolute stability criteria are derived. These conditions are given in the term of linear matrix inequalities (LMIs) and accordingly can be readily solved and checked. Finally, a numerical example is solved using the proposed method to demonstrate the effectiveness and its improvement over existing ones.

1. Introduction

Lur'e systems has received considerable attention since it was presented in 1940s, and a variety of issues about Lur'e systems has been investigated [1–4]. A time delay that frequently appear in practical systems may degrade the system performance and even cause the system to become unstable [5–9]. Thus, great efforts have been devoted to investigating the absolute stability of Lur'e systems with time-delay [10–13].

In [14,15], Han et al. investigate the absolute stability of a class of Lur'e systems, respectively, with constant delay and time-varying delay. In the case of time-varying delay, less conservative conditions are obtained in [16] by retaining some useful information and employing an improved free-matrix-weighting (IFMW) approach to consider the relationship between the time-varying and its upper bound. To reduce the conservativeness of stability analysis on Lur'e systems with constant delay, improved conditions were obtained by constructing a delay-partitioning Lyapunov–Krasovskii functional in [17]. Recently, a complete-partitioning approach was proposed to investigate the absolute stability of Lur'e systems with time-varying delay in [19], which significantly reduces the conservativeness. Nevertheless, the number and the dimension of LMIs involved in the condition increase sharply with the increase of the partitioning interval, which require higher computational cost. Therefore, how to reduce the computation burden and the conservativeness of the derived results need to be further investigated.

In this paper, we investigate the absolute stability of a class of Lur'e systems with time-varying delay and sector-bounded nonlinearity. By proposing an improved free-matrix-based inequality (FMBI) to bound the integral inequality yielded in the derivative of the Lyapunov–Krasovskii functional, new absolute stability conditions are presented based on the convex combination technique. Since the delay-partitioning technique is not involved, the derived results are relatively simple and less conservative than existing ones. The numerical example verifies the effectiveness and the merits of the presented method.

Notation: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $P>0$ means that the matrix P is symmetric and positive definite; diag$\{\cdots\}$ denotes a block-diagonal matrix; and the symmetric terms in a symmetric matrix are denoted by '$*$'; I is an appropriately dimensioned identity matrix; $Sym\{X\}=X+X^T$.

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2. Problem statement

Consider the following system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-d(t)) + Cw(t) \\ z(t) = Mx(t) + Nx(t-d(t)) \\ w(t) = -\varphi(t, z(t)) \\ x(t) = \phi(t), t \in [-h, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $w(t) \in \mathbb{R}^p$ and $z(t) \in \mathbb{R}^p$ are the state, input and output vectors of the system, respectively; A, B, C, M and N are constant matrices; the initial condition, $\phi(t)$, is a continuous vector-valued function of $t \in [-h, 0]$. $\varphi(t, z(t)) \in \mathbb{R}^p$ is a nonlinear function, which is continuous in t, globally Lipschitz in $z(t)$, and satisfies

$$[\varphi(t, z(t)) - K_1 z(t)]^T [\varphi(t, z(t)) - K_2 z(t)] \le 0 \tag{2}$$

for $\forall t \ge 0$, $\varphi(t, 0) = 0$, where K_1 and K_2 are real matrices and $\overline{K} = K_2 - K_1$ is a symmetric positive definite matrix. It is customarily said that the nonlinear function, $\varphi(t, z(t))$, belong to the sector $[K_1, K_2]$.

The delay, $d(t)$, is a differentiable function that satisfies

$$0 \le d(t) \le h \tag{3}$$

and

$$\mu_1 \le \dot{d}(t) \le \mu_2 \tag{4}$$

where h, μ_1 and μ_2 are constants.

Remark 1. By applying the loop transformation [20], we get that the absolute stability of system (1) in the sector $[K_1, K_2]$ is equal to that of the following system:

$$\begin{cases} \dot{x}(t) = (A - CK_1M)x(t) + (B - CK_1N)x(t-d(t)) + Cw(t) \\ z(t) = Mx(t) + Nx(t-d(t)) \\ w(t) = -\varphi(t, z(t)) \end{cases} \tag{5}$$

in the sector $[0, \overline{K}]$.

Before presenting our main results, we introduce the following lemmas, which are useful to derive the main results.

Lemma 1 ([18]). *Let x be a differentiable signal in $[\alpha, \beta] \to \mathbb{R}^n$. Then, for any symmetric matrices $R \in \mathbb{R}^{n\times n}$, X_1, $X_3 \in \mathbb{R}^{3n\times 3n}$, and matrices $X_2 \in \mathbb{R}^{3n\times 3n}$, N_1, $N_2 \in \mathbb{R}^{3n\times n}$ such that*

$$\Theta = \begin{bmatrix} X_1 & X_2 & N_1 \\ * & X_3 & N_2 \\ * & * & R \end{bmatrix} \ge 0 \tag{6}$$

the following inequality holds:

$$-\int_\alpha^\beta \dot{x}^T(s) R \dot{x}(s) ds \le \varpi^T \overline{\Omega} \varpi \tag{7}$$

where

$$\overline{\Omega} = (\beta - \alpha)(X_1 + \tfrac{1}{3} X_3) + \mathrm{Sym}\{N_1 G_1 + N_2 G_2\}$$

$$G_1 = \left[\overline{e}_1^T - \overline{e}_2^T\right]^T$$

$$G_2 = \left[2\overline{e}_3^T - \overline{e}_1^T - \overline{e}_2^T\right]^T$$

$$\overline{e}_1 = [I \;\; 0 \;\; 0]$$

$$\overline{e}_2 = [0 \;\; I \;\; 0]$$

$$\overline{e}_3 = [0 \;\; 0 \;\; I]$$

$$\varpi = \left[x^T(\beta) \;\; x^T(\alpha) \;\; \frac{1}{\beta - \alpha} \int_\alpha^\beta x^T(s) ds \right]^T.$$

For sake of the reduction of computation complexity, the following lemma is obtained by setting $X_1 = N_1 R^{-1} N_1^T$, $X_2 = N_1 R^{-1} N_2^T$, $X_3 = N_2 R^{-1} N_2^T$ in (7).

Lemma 2. *Let x be a differentiable signal in $[\alpha, \beta] \to \mathbb{R}^n$. Then, for any symmetric matrices $R(\in \mathbb{R}^{n\times n}) > 0$, and N_1, $N_2 \in \mathbb{R}^{3n\times n}$, the following inequality holds:*

$$-\int_\alpha^\beta \dot{x}^T(s) R \dot{x}(s) ds \le \varpi^T \hat{\Omega} \varpi \tag{8}$$

where

$$\hat{\Omega} = \mathrm{Sym}\{N_1 G_1 + N_2 G_2\} + (\beta - \alpha)(N_1 R^{-1} N_1^T + \tfrac{1}{3} N_2 R^{-1} N_2^T)$$

and $G_1, G_2, \overline{e}_1, \overline{e}_2, \overline{e}_3$ are defined in Lemma 1.

Lemma 3 ([21]). *Let D, E, and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t)F(t) \le I$. Then, for any scalar $\varepsilon > 0$*

$$DF(t)E + (DF(t)E)^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E.$$

3. Main results

In this section, we present our main results. To simplify vector and matrix representation, the followings are denoted:

$$\eta_1(t) = \left[x^T(t) \;\; \int_{t-d(t)}^t x^T(s) ds \;\; \int_{t-h}^{t-d(t)} x^T(s) ds \;\; x^T(t-d(t)) \right]^T$$

$$\eta_2(t) = \left[x^T(t) \;\; \dot{x}^T(t) \right]^T$$

$$\eta_3(t) = \left[x^T(t) \;\; x^T(t-d(t)) \;\; x^T(t-h) \right]^T$$

$$\eta_4(t) = \left[\frac{1}{d(t)} \int_{t-d(t)}^t x^T(s) ds \;\; \frac{1}{h-d(t)} \int_{t-h}^{t-d(t)} x^T(s) ds \right]^T$$

$$\xi(t) = \left[\eta_3^T(t) \;\; \eta_4^T(t) \;\; \dot{x}^T(t) \;\; \dot{x}^T(t-d(t)) \;\; w^T(t) \right]^T$$

$$e_i = \left[0_{n\times(i-1)n} \;\; I_n \;\; 0_{n\times(7-i)n} \;\; 0_{n\times p} \right], \quad i = 1, 2, \ldots, 7$$

$$e_8 = \left[0_{p\times 7n} \;\; I_p \right]$$

Now, we present the absolute stability criterion for system (1).

Theorem 1. *Given scalars h, μ_1 and μ_2, system (1) with a time-varying delay $d(t)$ satisfying (3) and (4) is absolutely stable in the sector $[K_1, K_2]$ if there exist matrices $P > 0$, $Q > 0$, $R > 0$, $Z > 0$, N_1, N_2, M_1, M_2, and a scalar $\varepsilon > 0$ such that the LMIs (9) and (10) are satisfied for $\dot{d}(t) \in \{\mu_1, \mu_2\}$*

$$\Phi_1 = \begin{bmatrix} \Xi|_{d(t)=h} & h\Pi_5^T N_1 & h\Pi_5^T N_2 \\ * & -hZ & 0 \\ * & * & -3hZ \end{bmatrix} < 0 \tag{9}$$

$$\Phi_2 = \begin{bmatrix} \Xi|_{d(t)=0} & h\Pi_8^T M_1 & h\Pi_8^T M_2 \\ * & -hZ & 0 \\ * & * & -3hZ \end{bmatrix} < 0 \tag{10}$$

where

$$\begin{aligned} \Xi = {} & \mathrm{Sym}\{\Pi_1^T P \Pi_2\} + \Pi_3^T Q \Pi_3 - (1 - \dot{d}(t)) \Pi_4^T Q \Pi_4 + e_1^T R e_1 - e_3^T R e_3 \\ & + h e_6^T Z e_6 + \mathrm{Sym}\{\Pi_5^T N_1 \Pi_6 + \Pi_5^T N_2 \Pi_7 + \Pi_8^T M_1 \Pi_9 \\ & + \Pi_8^T M_2 \Pi_{10} + \Pi_{11}^T \Pi_{12} \\ & - \varepsilon e_8^T e_8 - \varepsilon e_8^T \overline{K} M e_1 - \varepsilon e_8^T \overline{K} N e_2\} \end{aligned}$$

$$\Pi_1 = \left[e_1^T \;\; d(t) e_4^T \;\; (h - d(t)) e_5^T \;\; e_2^T \right]^T$$

$$\Pi_2 = \left[e_6^T \;\; e_1^T - (1 - \dot{d}(t)) e_2^T \;\; (1 - \dot{d}(t)) e_2^T - e_3^T \;\; (1 - \dot{d}(t)) e_7^T \right]^T$$

$$\Pi_3 = \left[e_1^T \;\; e_6^T \right]^T$$

$$\Pi_4 = \left[e_2^T \;\; e_7^T \right]^T$$

$$\Pi_5 = \left[e_1^T \;\; e_2^T \;\; e_4^T \right]^T$$

$$\Pi_6 = \left[e_1^T - e_2^T \right]^T$$