Fuzzy Inf. Eng. (2016) 8: 75–85

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ORIGINAL ARTICLE

Boundary and Interior Nodes in a Fuzzy Graph Using Sum Distance

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Mini Tom · M.S. Sunitha

Received: 7 February 2015/ Revised: 7 May 2015/
Accepted: 6 July 2015/

Abstract In this paper, the concepts of boundary nodes and interior ones are introduced in a fuzzy graph based on sum distance with relationship among boundary nodes, interior nodes, fuzzy cutnodes and complete nodes. It is observed that fuzzy cutnodes can be boundary nodes and there are nodes in G, neither boundary nodes nor interior ones. It is verified that a complete node need not be a boundary one and a node which is a boundary one of all other nodes need not be complete. In fuzzy trees, it is observed that fuzzy end nodes need not be boundary ones and vice versa. It is verified that in a complete fuzzy graph there exist at most one node which is not a boundary one. Also boundary nodes of a self centered fuzzy cycle are identified together with interior nodes of a complete fuzzy graph.

Keywords Fuzzy graph · Sum distance · Fuzzy cycle · Boundary · Interior

1. Introduction

Zadeh introduced his landmark paper "Fuzzy Sets" [1] in the year 1965. Ten years after this remarkable paper, theory of fuzzy graphs was independently developed by

Mini Tom (✉)
Department of Mathematics, SCMS School of Engineering and Technology, Karukutty - 683582, Kerala, India
email: minitom2001@yahoo.com
M.S. Sunitha
Department of Mathematics, National Institute of Technology Calicut, Kozhikode - 673601, Kerala, India
Peer review under responsibility of Fuzzy Information and Engineering Branch of the Operations Research Society of China.

http://dx.doi.org/10.1016/j.fiae.2015.07.001

Rosenfeld [2], Yeh and Bang [3]. Akram et al. introduced the concepts of bipolar fuzzy graphs and interval-valued fuzzy line graphs [4-8]. The author further defined its length, distance, eccentricity, radius and diameter and introduced the concept of self centered bipolar fuzzy graphs [9]. He also introduced the concept of an antipodal intuitionistic fuzzy graph and self median intuitionistic fuzzy one of a given intuitionistic fuzzy one [10]. Fuzzy competition graph was introduced by Samanta and Pal [11]. Two generalizations of fuzzy competition graph as fuzzy k-competition graphs and p-competition fuzzy graphs are also defined by the same authors. In [12] Samanta et al. defined another generalization of fuzzy competition graph, called an m-step competition one. Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties. He introduced the concept of μ-distance in fuzzy graphs and based on this μ-distance, Bhattacharya [13] has introduced concepts of eccentricity and center in fuzzy graphs and the properties of this metric are further studied by Sunitha and Vijyakumar [14]. Bhutani and Roesnfeld have advanced the concepts of strong arcs [15], fuzzy end nodes [16] and geodesics in fuzzy graphs [17]. Further studies based on the g-distance are carried out by Sameena and Sunitha in [18] and [19]. Chartrand and Zang [20] introduced the concepts of boundary vertex, interior vertex, boundary and interior based on geodesic distance in crisp graph. Based on g-distance, Linda and Sunitha introduced the concepts of g-peripheral nodes, g-boundary nodes and g-interior nodes [21]. Nagoorgani and Umamaheswari developed the concept of fuzzy detour μ-distance and defined fuzzy detour μ-center and studied its properties [22]. Fuzzy detour g-distance was introduced by Linda and Sunitha in [23]. The authors discussed the concept of fuzzy detour g-boundary nodes and fuzzy detour g-interior ones in fuzzy graphs [24]. Tom and Sunitha [25] have defined length of any $u-v$ path P as sum of weights of the arcs in P and defined the distance between u and v called the sum distance denoted by $d_s(u,v)$. Based on this metric the concepts are studied in eccentricity, radius, diameter, center, self centered fuzzy graphs etc. and the properties are also obtained in eccentric nodes, peripheral ones, central ones [25].

Detection of nodes on the network boundary is necessary for correct operation in many wireless applications. Awareness of boundary prevents the processing of requests that are impossible to satisfy the fact when the request lies outside the network space. Moreover, nodes close to network boundary are often assumed to provide the best candidate for beacon nodes in virtual co-ordinate construction. Focussing on these applications a research is carried out for boundary nodes and interior ones in fuzzy graph based on sum distance in this paper.

Section 2 contains preliminaries. In Section 3, boundary node in a fuzzy graph based on sum distance is defined and verified that a complete node need not be a boundary node and a node, a boundary node of all other ones, need not be complete. It is observed that in fuzzy graphs fuzzy cutnodes can be boundary nodes. Also the boundary nodes of a complete fuzzy graph are identified in this section. Interior nodes and interior of a fuzzy graph based on sum distance is defined in Section 4. It is observed that in a fuzzy graph there are nodes which are neither boundary nodes nor interior ones. It is verified that in a complete fuzzy graph there exists at most one