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Causal effect identification in acyclic directed mixed graphs and gated models ☆

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ABSTRACT

We introduce a new family of graphical models that consists of graphs with possibly directed, undirected and bidirected edges but without directed cycles. We show that these models are suitable for representing causal models with additive error terms. We provide a set of sufficient graphical criteria for the identification of arbitrary causal effects when the new models contain directed and undirected edges but no bidirected edge. We also provide a necessary and sufficient graphical criterion for the identification of the causal effect of a single variable on the rest of the variables. Moreover, we develop an exact algorithm for learning the new models from observational and interventional data via answer set programming. Finally, we introduce gated models for causal effect identification, a new family of graphical models that exploits context specific independences to identify additional causal effects.

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1. Introduction

Undirected graphs (UGs), bidirected graphs (BGs), and directed and acyclic graphs (DAGs) have extensively been studied as representations of independence models. DAGs have also been studied as representation of causal models, because they can model asymmetric relationships between random variables. DAGs and UGs (respectively BGs) have been extended into chain graphs (CGs), which are graphs with possibly directed and undirected (respectively bidirected) edges but without semidirected cycles. Therefore, CGs can model both symmetric and asymmetric relationships between random variables. CGs with possibly directed and undirected edges may represent a different independence model depending on whether the Lauritzen–Wermuth–Frydenberg (LWF) or the Andersson–Madigan–Perlman (AMP) interpretation is considered [16,1]. CGs with possibly directed and bidirected edges have a unique interpretation, the so-called multivariate regression (MVR) interpretation [8]. MVR CGs have been extended by (i) relaxing the semidirected acyclity constraint so that only directed cycles are forbidden, and (ii) allowing up to two edges between any pair of nodes. The resulting models are called original acyclic directed mixed graphs (oADMGs) [25]. AMP CGs have also been extended similarly [19]. The resulting models are called alternative acyclic directed mixed graphs (aADMGs).

In this paper, we combine oADMGs and aADMGs into what we simply call ADMGs. These are graphs with possibly directed, undirected and bidirected edges but without directed cycles. Moreover, there can be up to three edges between any pair of nodes. This work complements the existing works for the following reasons. To our knowledge, the only mixed

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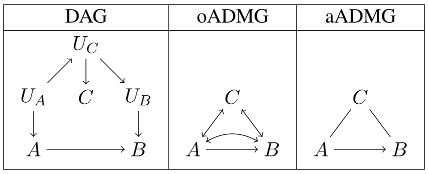

Fig. 1. Example where $p(B|\widehat{A})$ is identifiable from the aADMG but not from the oADMG.

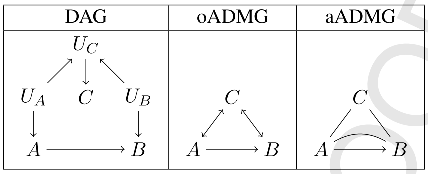

Fig. 2. Example where $p(B|\widehat{A})$ is identifiable from the oADMG but not from the aADMG.

graphical models in the literature that subsume AMP CGs are the already mentioned aADMGs and the so-called marginal AMP CGs [18]. However, marginal AMP CGs are simple graphs with possibly directed, undirected and bidirected edges but without semidirected cycles and, moreover, some constellations of edges are forbidden. Therefore, marginal AMP CGs do not subsume ADMGs. Likewise, no other family of mixed graphical models that we know of (e.g. oADMGs, summary graphs [8], ancestral graphs [26], MC graphs [15] or loopless mixed graphs [27]) subsume AMP CGs and hence ADMGs. To see it, we refer the reader to the works by Richardson and Spirtes [26, p. 1025] and Sadeghi and Lauritzen [27, Section 4.1].

In addition to represent independence models, some of the families of graphical models mentioned above have been used for causal effect identification, i.e. to determine if the causal effect of an intervention is computable from observed quantities. For instance, Pearl's approach to causal effect identification makes use of oADMGs to represent causal models over the observed variables [22]. The directed edges represent potential causal relationships, whereas the bidirected edges represent potential confounding, i.e. a latent common cause. A key feature of Pearl's approach is that no assumption is made about the functional form of the causal relationships. That is, each variable A is an unconstrained function of its observed causes $Pa(A)$ and its unobserved causes U_A, i.e. $A = f(Pa(A), U_A)$. In this paper, we study causal effect identification under the assumption that $A = f(Pa(A)) + U_A$, i.e. under the assumption of additive errors. This is a rather common assumption in causal discovery, e.g. see [7,14,23]. Specifically, we show that ADMGs are suitable for representing such causal models: An undirected edge between two nodes represents potential dependence between their error terms given the rest of the error terms, as opposed to a bidirected edge that represents potential marginal dependence due to confounding. The reason for studying ADMGs for causal effect identification is that we may identify more causal effects from them than from oADMGs, since the former are tailored to the additive error assumption. We illustrate this question with the example in Fig. 1, which is borrowed from Peña [19]. The ADMGs in the figure represent the causal model over the observed variables represented by the DAG. The oADMG is derived from the DAG by keeping the directed edges between observed variables, and adding a bidirected edge between two observed variables if and only if they have a confounder [35, Section 5]. The aADMG is derived from the DAG by keeping the directed edges between observed variables, and adding an undirected edge between two observed variables if and only if their unobserved causes are not separated in the DAG given the unobserved causes of the rest of the observed variables. Clearly, the effect on B of an intervention on A, i.e. $p(B|\widehat{A})$, is not identifiable from the oADMG [22, p. 94], but it is identifiable from the aADMG and is given by

$$p(B|\widehat{A}) = \sum_c p(B|A,c)p(c).$$

To see it, recall that we assume additive noise. This implies that C determines U_C, which blocks the path $A \leftarrow U_A \rightarrow U_C \rightarrow U_B \rightarrow B$ in the DAG. This can also be seen directly in the aADMG, as C blocks the path $A - C - B$. Therefore, we can identify the desired causal effect by just adjusting for C, since C blocks all non-causal paths from A to B. It is worth mentioning that there are also cases where the oADMG allows for causal effect identification whereas the aADMG does not. One such case is shown in Fig. 2, where we have just replaced the edge $U_C \rightarrow U_B$ in Fig. 1 with the edge $U_C \leftarrow U_B$. Specifically, $p(B|\widehat{A})$ is not identifiable from the aADMG by Theorem 12 in this article, whereas it is identifiable from the oADMG, i.e. $p(B|\widehat{A}) = p(B|A)$. Therefore, oADMGs and aADMGs are more complementary than competing causal models. To further illustrate our point, we make the example in Fig. 1 more concrete by turning it into the following invented gambling game:

$$U_A \sim N(0, \sigma)$$
$$U_C \sim N(U_A, \sigma)$$
$$U_B \sim N(U_C, \sigma)$$

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