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Structured probabilistic rough set approximations ☆

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ABSTRACT

Probabilistic rough set approximations are proposed based on a conditional probability to describe the desired levels of precision between the equivalence classes and an approximated set. This definition shows the detailed information on individuals satisfying some conditions but ignores the structural information. In this paper, applying the structured granules in a coarsened-grained universe, we introduce structured probabilistic rough set approximations between the power sets of the original universe and the coarsened-grained universe. By using the zooming-in and structured probabilistic rough approximation operators, two pairs of probabilistic rough lower and upper approximations on the same universe are investigated. Related properties and relationships of them are investigated. Furthermore, applying the Bayesian decision procedure, conditional probability and loss functions, three-way classifications in structured probabilistic rough set approximations are then proposed to classify the structured granules of the coarsened-grained universe into three disjoint structured probabilistic regions. This method gives the values of the pair of thresholds. Meanwhile, by using the minimum-risk decision rules, we also can construct the structured probabilistic rough lower and upper approximations. Finally, we discuss the monotonicity of structured probabilistic positive and negative regions.

1. Introduction

Pawlak's rough set theory [25] introduces the lower and upper approximations based on the totally inclusion or non-empty overlapping between the equivalence classes and an approximated subset. Many researchers pay their attentions to the extensions of Pawlak's rough set theory [3,13,28,34,36,44–46,54]. In Pawlak's rough set theory, an object belongs to the Pawlak lower approximation of an approximated set if and only if the equivalence class including the object must be a subset of the set. This requirement is too rigid [51]. In order to resolve this question, Pawlak and Wong proposes probabilistic rough sets based on the threshold 0.5 [27]. Yao introduces probabilistic rough set models based on a pair of thresholds [43]. And Ziarko investigates variable threshold rough sets by using a relative degree of misclassification function [57]. Then based on a pair of thresholds and a conditional probability, Yao gives a unified framework of probabilistic rough sets [49]. That is to say, probabilistic rough sets show a classification which is not fully correct nor certain, but with a certain level of tolerance for error [50]. Furthermore, probabilistic rough sets are then extended to different probabilistic rough set models [8,14,19,23,24,33,35,38,39,42,58].

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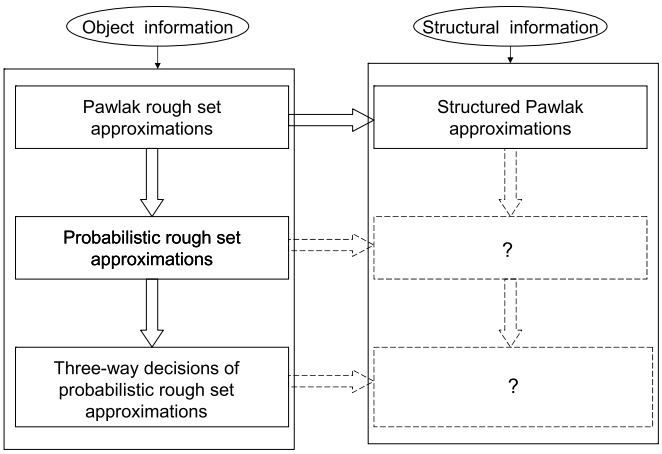

Fig. 1. The developments of rough set approximations.

Three-way decision theory is proposed by Yao on the basis of the study of rough set [25,26] and decision-theoretic rough set [43]. It is constructed based on the notions of acceptance, rejection and non-commitment under an evaluation function. The early idea of it is first proposed by Yao in [47]. Then the deeply developments of this theory are made by Herbert and Yao [9,50]. Three-way decisions can be used to rationally interpret the three regions in rough set [50]. Corresponding to the three regions: positive, negative and boundary regions in rough set, three-way decision theory shows the regions of acceptance, rejection and non-commitment in a ternary classification. Meanwhile, this theory provides the mainly results of probabilistic rough sets [50–52]. An analysis of three-way decision rules in the classical rough set model and the decision-theoretic rough set model is also investigated [37,41]. Three-way decisions provide a close relationship between rough set and Bayesian decision procedure [35]. It has been shown that probabilistic three-way decisions are superior to both Pawlak three-way decisions and two-way decisions [51]. And the pair of thresholds in probabilistic rough set models can be systematically obtained based on minimum costs or risks of various decisions. In recent years, more and more papers pay much attention to the generalizations and applications of three-way decisions [4,7,10–12,15,17,18,20, 29–31,40,55,56].

Pawlak rough set approximations and probabilistic rough set approximations show the detailed information of objects satisfying the conditions between the equivalence classes and an approximated set but ignore the structural information. Shafer proposes the outer and inner reductions in the study of belief functions by using the equivalence classes and introduces an operator to refine the coarsened-grained universe [32]. Applying them Dubois and Prade give birth to the fuzzy rough sets and rough fuzzy sets [5]. Since the decomposition of the unions of equivalence classes in Pawlak rough lower and upper approximations is unique, Bryniarski proposes the lower and upper approximations as families of equivalence classes [1]. These approximations are called structured Pawlak approximations [53] since they retain the structural information. Following the zooming-in operator [32] and structured Pawlak approximation operators [1], Yao gives a partition model of granular computing in a set-theoretic setting, and discussed the coarsening and refinement between an original universe and a coarsened-grained universe [48]. Ma extends this result, and investigates a covering model of granular computing based on a reflexive binary relation [21]. Furthermore, Ma redefines the zooming-in operator under a reflexive binary relation, and discussed a covering model of granular computing based on the zooming-in operator and the lower and upper approximations in a coarsened-grained universe [22]. All these lower and upper approximations regard each equivalence class as a whole, and require the completely inclusion or non-empty overlapping between the equivalence classes and an approximated set, which is too rigid to realize. How to resolve this question? The left column in Fig. 1 shows the developments of rough set approximations, which describe the object information in the original universe. Meanwhile, in the first row, Pawlak rough set approximations and structured Pawlak approximations have different slightly form. It is worth to note that structured Pawlak approximations includes structural information. This is the approach to resolve the question above, and give the developments of structured Pawlak approximations in the right column of Fig. 1 with structural information.

The main objective of this paper is to introduce the idea of probabilistic rough set approximations into structured Pawlak approximations, and investigate their structural information. Applying the conditional probability and a pair of thresholds, we can put the desired levels of precision between the equivalence classes and an approximated set with maintaining the structural information. This leads to structured probabilistic rough set approximations between an original universe and the corresponding coarsened-grained universe. Since the zooming-in operator expands any structured granules in a coarsened-grained universe to a subset of the original universe, this operator with the structured probabilistic rough set approximation operators can be used to construct two pairs of probabilistic rough set approximations on different universes, and reveal the relationships between the original universe and the coarse-grained universe. In order to obtain the pair of thresholds representing the desired levels of precision between an equivalence class and an approximated set, Bayesian

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