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Inference procedures and engine for probabilistic argumentation

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A B S T R A C T

Probabilistic Argumentation (PA) is a recent line of research in AI aiming to combine the strengths of argumentation and probabilistic reasoning. Though several models of PA have been proposed, the development of practical applications is still hindered by the lack of inference procedures and reasoning engines. In this paper, we present a reduction method to compute a recently proposed model of PA called PABA. Using the method we design inference procedures to compute the credulous semantics, the ideal semantics and the grounded semantics for a general class of PABA frameworks, that we refer to as Bayesian PABA frameworks. We also show that, though restricting to Bayesian PABA frameworks, the inference procedures can be used to compute other PA models thanks to simple translations. Finally, we implement the inference procedures to obtain a multi-semantics engine for probabilistic argumentation and demonstrate its usage.

1. Introduction

An Abstract Argumentation (AA) framework is a pair (AR, Att) where AR is a set of arguments, $Att \subseteq AR \times AR$ is a set of attacks between arguments [16]. The semantics of AA rest on Dung's crisp notion of argument *acceptability*, namely an argument X is acceptable wrt a set S of arguments iff S attacks every argument attacking X. Over the last decade, AA has been used to unify different reasoning formalisms in AI, and also extended in several directions to address its shortcomings. Notably, Probabilistic Argumentation (PA) extends AA with classical probability theory to deal with a probabilistic distribution of AA frameworks representing different "possible worlds". Several PA models have been proposed. On the abstract level there are Dung and Thang's model [19] (for short, DT's PAA) among others [31,40,28,23,36]. On the instantiated level, there are Probabilistic Assumption-based Argumentation (PABA [19]) which instantiates DT's PAA with Assumption-based Argumentation (ABA [10]); p-ASPIC [38] which also instantiates DT's PAA but using (a simplified version of) ASPIC [37]. In this paper we are interested in computing PA semantics. Noting that many existing PA models can be translated into PABA, we first focus on computing PABA semantics. To motivate our work, let's construct a sample PABA framework[1] for a story line used in [38] to motivate p-ASPIC.[2]

Example 1. Suppose you are planing a road trip with four friends to the beach: Anne, Bob, Chris, David. You believe that: 1) Anne surely wants to go, however Bob, Chris, David just probably, with probabilities 0.5, 0.6 and 0.7 respectively; 2) when

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[1] PABA is formally defined in section 2.
[2] A general translation of p-ASPIC into PABA is presented in section 8.

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a friend wants to go, then if possible she or he will join the trip; 3) if Chris and David join then Anne will not join because both Chris and David are in love with her; 4) your car seats at most three passengers. So your beliefs can be represented by a PABA framework $\mathcal{P} = (\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ where

- $\mathcal{A}_p = \{p\_bw, p\_cw, p\_dw\}$ consists of so-called *probabilistic assumptions* representing your uncertain beliefs about whether Bob, Chris, David want to go; and $\mathcal{R}_p$ consists of so-called *probabilistic inference rules* representing the probabilities of probabilistic assumptions.[3]

$$[p\_bw : 0.5] \leftarrow \qquad [p\_cw : 0.6] \leftarrow \qquad [p\_dw : 0.7] \leftarrow$$

- $\mathcal{F} = (\mathcal{R}, \mathcal{A}, \overline{})$ is an ABA framework where
  - $\mathcal{R}$ consists of inference rules $r_1, \ldots, r_{10}$ for:
    - representing (1): $r_1 : aw \leftarrow \quad r_2 : cw \leftarrow p\_cw \quad r_3 : bw \leftarrow p\_bw \quad r_4 : dw \leftarrow p\_dw$
    - representing (2):

$$r_5: \ aj \leftarrow aw, arguably(aj), \lceil r_5 \rceil \qquad r_7: \ cj \leftarrow cw, arguably(cj), \lceil r_7 \rceil$$
$$r_6: \ bj \leftarrow bw, arguably(bj), \lceil r_6 \rceil \qquad r_8: \ dj \leftarrow dw, arguably(dj), \lceil r_8 \rceil$$

      where $aj, \ldots, dj$ stand for **A**nne **j**oins, ..., **D**avid **j**oins; and $aw, \ldots, dw$ stands for **A**nne **w**ants to go to the beach, ..., **D**avid **w**ants to go to the beach; and $\lceil . \rceil$ maps a defeasible rule r_i into a sentence $\lceil r_i \rceil$ of the underlying language indicating the applicability of r.
    - representing (3): $r_9 : \ \neg\lceil r_5 \rceil \leftarrow cj, dj$
    - representing (4): $r_{10} : \ false \leftarrow aj, bj, cj, dj$[4]
  - $\mathcal{A} = \{\lceil r_5 \rceil, \lceil r_6 \rceil, \lceil r_7 \rceil, \lceil r_8 \rceil\} \cup \{arguably(l) \mid l \in \{aj, bj, cj, dj\}\}$ is a set of assumptions.
  - $\overline{}$ maps assumptions in $\mathcal{A}$ to their contraries: $\overline{\lceil r_i \rceil} = \neg\lceil r_i \rceil$ and $\overline{arguably(l)} = \neg l$.

The semantics of DT's PAA and PABA can be seen respectively as probabilistic versions of AA semantics and ABA semantics. Concretely, an AA semantics (resp. ABA semantics) *sem* induces a DT's PAA semantics (resp. a PABA semantics) mapping each argument X (resp. each proposition π) to the probability that X (resp. some argument for π) is accepted under *sem*, denoted $Prob_{sem}(X)$ (resp. denoted $Prob_{sem}(\pi)$). Since ABA is just an instance of AA, we can also say that each AA semantics *sem* induces a PABA semantics $Prob_{sem}(.)$. For example, AA **gr**ounded semantics [16] (the most skeptical) induces PABA grounded semantics $Prob_{gr}(.)$; AA **cr**edulous (aka admissible) semantics [16] (the least skeptical) induces PABA credulous semantics $Prob_{cr}(.)$; while AA ideal semantics [18] (the ideally skeptical) induces PABA ideal semantics $Prob_{id}(.)$. It turns out that $Prob_{gr}(.)$ and $Prob_{cr}(.)$ identify faithfully the **lower** bound of probability and the **upper** bound of probability (while $Prob_{id}(.)$ identifies the **ideal** point in this probability interval). For example, wrt the PABA framework in Example 1, $Prob_{gr}(aj) = 0.58$ and $Prob_{cr}(aj) = 0.79$ are respectively the lower bound probability and the upper bound probability of aj. As the numbers do not seem to come trivially, let's explain them. There are eight possible worlds representing the actual wants of Bob, Chris and David. For example possible world $\omega_0 = \{p\_bw, p\_cw, p\_dw\}$ represents that all three friends want to go to the beach, whereas $\omega_1 = \{\neg p\_bw, p\_cw, p\_dw\}$ represents that Chris and David want to go but not Bob. Anne *surely* does not join if both Chris and David want to join but not Bob (i.e. $\neg bw \wedge cw \wedge dw$), which happens in possible world ω_1, because the rule $\neg\lceil r_5 \rceil \leftarrow cj, dj$ fires. However, if Bob, Chris and David all want to join (i.e. $bw \wedge cw \wedge dw$), which happens in ω_0, then Anne may or may not join. When she joins, she will join with Bob, and with either Chris or David because the car cannot seat four passengers (note that $\neg\lceil r_5 \rceil \leftarrow cj, dj$ does not fire in this situation). And when she does not join, then the three passengers joining the trip are Bob, Chris and David. In the remaining possible worlds $\omega_2, \omega_3, \ldots, \omega_7$, either Chris or David does not join and hence Anne surely joins. Hence, the upper bound probability that Anne joins is the one's complement of the probability of ω_1 (i.e. $1 - 0.5 \times 0.6 \times 0.7 = 0.79$), while the lower bound probability that Anne joins equals the upper bound probability subtracting the probability of ω_0 (i.e. $0.79 - 0.5 \times 0.6 \times 0.7 = 0.58$).

To the best of our knowledge, PABA inference procedures haven been unexplored so far. Note that PABA inference procedures subsume ABA proof procedures since an ABA framework $\mathcal{F}$ can be seen as a PABA framework $(\mathcal{A}_p, \mathcal{R}_p, \mathcal{F})$ with empty sets $\mathcal{A}_p$ and $\mathcal{R}_p$. This also suggests that, to develop PABA inference procedures, one can (and should) reuse existing ABA proof procedures [17,18,41,13], rather than develop them from scratch. Interestingly this is fulfilled if one can afford to explore all possible worlds. Concretely, to compute $Prob_{sem}(\pi)$, one can iterate over all possible worlds, using ABA proof procedures to pick out those in which π is accepted, then returns the sum of the probabilities of such worlds. For example, to compute $Prob_{cr}(aj)$, one iterates over eight possible worlds $\omega_0, \omega_1, \ldots \omega_7$, picking out $\omega_1, \ldots \omega_7$, then returns $P(\omega_1) + \cdots + P(\omega_7)$. Unfortunately this "naive" approach *always* results in an exponential blowup since in the first step alone, it has as many as $2^{|\mathcal{A}_p|}$ possible worlds to consider.

[3] It is sound to equate probabilistic assumptions with binary random variables of the standard probabilistic terminologies.

[4] Shorthand for several transpositions such as $\neg aj \leftarrow bj, cj, dj$.