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Formal methods for reasoning and uncertainty reduction in evidential grid maps

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ABSTRACT

Information fusion is the task of combining data collected from different sources into a unified representation. Here, a main challenge is to deal with the inherent uncertainty contained in the information, such as sensor noise, conflicting information, or incomplete knowledge. In current approaches, one usually employs independence assumptions in order to reduce the complexity. Because of this, the full potential of the gathered data is often not fully exploited and the fusion may lead to additional uncertainty. In order to reduce this uncertainty, further information in form of background and expert knowledge can be utilized, which is often available for real-world scenarios. However, reasoning on this knowledge is a computational complex task. In this work, we propose a methodology which utilizes formal methods for that reasoning, which allows to relax some of the independence assumptions. We demonstrate the proposed methodology using evidential grid maps – a belief function-based environment representation, in which different kinds of uncertainty are represented explicitly. Our methodology is evaluated based on basic structures as well as on real-world data sets. The results show that the uncertainty in the maps is significantly reduced by considering dependencies among cells.

1. Introduction

When a technical system acts in or interacts with an environment, it relies on a suitable representation of this environment. In many practical applications, the required information is provided by multiple sources, such as sensory data or expert knowledge. Additionally, information fusion techniques [1] are usually needed to create a unified representation. One of the largest research fields on this topic is mobile robotics [2], where the robot collects data using its sensors and builds a spatial representation of its environment.

In the past decades, many different representation forms have been proposed and even more approaches were developed to build these from scratch [3]. The algorithms have to evaluate the data from different sources with respect to their uncertainty (e.g. sensor noise, conflicting information, and incomplete or missing knowledge) and, afterwards, derive the most accurate representation from it, which is compatible with all provided information. Obviously, this is a complex task, since practically relevant environment sizes reach from tens to thousands of meters and typical sensors provide a huge amount of data every second. Both lead to a large combinatorial complexity. In order to make the computation feasible, most approaches use restrictive independence assumptions, such as conditional independencies of particular parts in the

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environment. As a consequence, these algorithms cannot exploit the full potential of the data (later, Section 3 illustrates and discusses this in more detail).

In this work,[1] we propose the exploitation of formal methods to address this issue. Formal methods are well-known for their capabilities to efficiently traverse and prune large search spaces. In particular, we exploit the deductive power of *maximum satisfiability solvers* (MAX-SAT, [5,6]) in order to consider dependencies among different parts of the environment, which are dropped by standard approaches for the reasons discussed above. This allows us to use expert knowledge about the environment to reduce the uncertainty in the derived results and to infer the true state of the corresponding parts of the environment.

We apply the proposed methodology to occupancy grid maps [7], a popular spatial representation in robotics. In particular, we are using evidential grid maps [8], which are based on the belief function theory [9] and are well suited for navigation tasks [10,11]. Furthermore, they explicitly represent different kinds of uncertainty [12], which provide additional information to our methodology that is not present in probabilistic grid maps. We demonstrate how the proposed methodology reduces the uncertainty by using expert knowledge about the environment. In particular, we apply our approach in the context of evidential grid maps representing office environments. More precisely, a set of hand-crafted benchmarks containing basic room structures as well as two real-world maps are used as benchmarks in our evaluations.

The reminder of this paper is structured as follows: In Section 2, we review the background on evidential grid maps and information fusion using this kind of maps. In Section 3, the limitations of the current approaches are discussed and a motivation for the proposed methodology is provided. In Section 4, we present our general idea, while in Section 5, we describe the solution in detail. In Section 6, the results of an empirical evaluation using small structures as well as real-world datasets are presented and discussed. The paper concludes with a summary and an outlook.

2. Background

This section reviews the background on occupancy grid maps in general and evidential grid maps in particular. Afterwards, the corresponding fusion process is re-visited, which is applied in order to generate a more precise grid map from the combination of data. Finally, we present how we convert evidential grid maps into a categorical representation for using them as input for the methodology proposed in this work. In order to ease the descriptions, we keep the considered models as simple as possible.

2.1. Evidential grid maps

One particular and frequently used spatial environment representation are *occupancy grid maps* [7], which distinguishes between empty and occupied areas in the environment.

Definition 1. An *occupancy grid map* represents a spatial environment in terms of a discretized grid, where each grid cell may either be empty (denoted by e) or occupied (denoted by o).

The information on whether a cell is empty or occupied is obtained from different sources, e.g. gathered by sensors, prior knowledge, etc. Because these sources of information may be afflicted with uncertainty (e.g. sensor noise, contradictory sensor measurements among different sensors or over time, vague expert knowledge, or the simple non-availability of information), a formalism to represent uncertain information in a given map is required. To this end, the state of a cell is usually modeled probabilistically with a single occupancy probability distribution $P(o)$. Here, for example, $P(o) = 0.3$ represents that the corresponding cell in the grid may be occupied with probability of 30%, what implies that it may be empty with probability of 70% ($P(e) = 0.7$), since the probability theory requires that $P(o) + P(e) = 1$.

The proposed methodology, however, benefits from an explicit representation of different dimensions of uncertainty, since e.g. the complete lack of information cannot explicitly be expressed by probabilities. A uniform distribution ($P(e) = 0.5$ and $P(o) = 0.5$) could work, but bears the risk of being misinterpreted with the fact that the cell is considered to be empty/occupied with probability of 50% due to conflicting measurements. Therefore, the belief function theory [9,13], which is often considered as a generalization of the well-known Bayesian probability theory, is used in this work. This theory allows us to assign belief mass not only to the singletons of a hypotheses space (here e and o), but also to all subsets including $\{e, o\}$ and $\emptyset$. The belief mass is assigned using so-called *mass functions*, which are defined as follows.

Definition 2. Let Θ be the frame of discernment, i.e. the hypotheses space, and $A \subseteq \Theta$ a hypothesis of Θ. Then, a *mass function* is a mapping $m : \mathcal{P}(\Theta) \to [0, 1]$ assigning a mass value to each hypothesis A of Θ such that[2]

$$\sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

[1] This work extends a previous conference paper [4].

[2] Note that we are using unnormalized mass functions here, which allow $m(\emptyset) \neq 0$.