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Uncertain lightweight ontologies in a product-based possibility theory framework ☆

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This paper investigates an extension of lightweight ontologies, encoded here in DL-Lite languages, to the product-based possibility theory framework. We first introduce the language (and its associated semantics) used for representing uncertainty in lightweight ontologies. We show that, contrarily to a min-based possibilistic DL-Lite, query answering in a product-based possibility theory is a hard task. We provide equivalent transformations between the problem of computing an inconsistency degree (the key notion in reasoning from a possibilistic DL-Lite knowledge base) and the weighted maximum 2-Horn SAT problem. The last part of the paper provides an encoding of the problem of computing inconsistency degree in product-based possibility DL-Lite as a weighted set cover problem and the use of a greedy algorithm to compute an approximate value of the inconsistency degree. This encoding allows us to provide an approximate algorithm for answering instance checking queries in product-based possibilistic DL-Lite. Experimental studies show the quality of the approximate algorithms for both inconsistency degree computation and instance checking queries.

1. Introduction

Knowledge representation for the semantic web requires an analysis of the universe of discourse in terms of concepts, definitions, objects, roles, etc., and then selecting a computer-usable version of the results. In this context, ontologies play an important role for the success of the semantic web as they provide shared vocabularies for different domains, such as medicine and bio-informatics among others. There are many representation languages for ontologies. Among them, description logics (DLs) [4,45] provide solid theoretical foundations to ontologies thanks to their clear semantics and formal properties. Besides, despite its syntactical restrictions, the DL-Lite family [2,3,58,57] enjoys good computational properties while still offering interesting capabilities in representing terminological knowledge. That is why a large amount of works has been recently dedicated to this family and this paper is a contribution to this general research line.

The dynamics of information available on the web naturally leads to a continuous evolution of the ontologies and to a permanent need to merge or to align them. As a result, we are often confronted with uncertainty in the used information. Hence, it is crucial to extend DL-Lite in order to represent and handle uncertain pieces of information. Many approaches

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have been proposed to extend description logics (see the related works section). Some of these extensions have been done in a probability theory framework [26,24,44,42]. Other extensions of description logics use fuzzy set theory (e.g., [51,53,11,12,40,56]). Besides, the extensions based on a possibility theory framework have also received a lot of attention (see e.g. [22,37,43]). Among these extensions, the min-based possibilitic DL-Lite has been proposed in [5]. It has been shown that the min-based possibilistic DL-Lite is a simple extension of DL-Lite and it is done without extra computational cost. Hence, query answering and computing conflicts are tractable and efficient in min-based possibilitic DL-Lite.

In fact, there are two major definitions of a possibility theory: min-based (or qualitative) possibility theory and product-based (or quantitative) possibility theory [18]. At the semantic level, these two theories share the same definitions, including the concepts of possibility distributions, necessity measures, possibility measures and the definition of normalized possibility distributions. However, they differ in the way they define conditioning and also in the way possibility degrees (or compatibility degrees) are defined over interpretations (or solutions). Min-based possibility theory is appropriate when the uncertainty scale only encodes a plausibility ordering (a total pre-order) over interpretations or formulas. Product-based possibility theory is appropriate when uncertainty degrees represent degrees of surprise or reflect the result of transforming a probability distribution into a possibility distribution. Hence, in the context of DL-Lite, if uncertainty degrees represent a plausibility encoding between assertions (a total pre-order), then min-based possibilistic DL-Lite should be used. Now, if the uncertainty degree represents a degree of surprise in the sense of Spohn's Ordinal Conditional Functions (OCF) [46,47] or a result of transforming a probability distribution into a possibility distribution [20], then product-based possibility distribution DL-Lite is more appropriate.

In fact, Spohn when he introduced the concept of kappas functions (or conditional ordinal functions), this was done in the context of belief change. Indeed, in his paper [46], Spohn not only gave foundations of ordination conditional functions (OCF) but he also introduced different strategies for belief changes. Since ontologies and Aboxes change dynamically, it is then natural and important to explore the use of kappas calculus (or OCF) and possibility theory for handing inconsistent and dynamic ontologies and Aboxes.

Beside, Spohn's ordinal conditional functions, or possibility theory frameworks, offer interesting alternatives for representing uncertainty degrees that do not represent exact probabilities. These uncertainty degrees may represent degrees of surprise or simply an ordering on available information. They may also represent qualitative or abstract probabilities (see for instance [17]) or even infinitesimal probabilities ([41,1]).

Now, in the context of uncertain ontologies and uncertain assertional facts, it is not always the case that uncertainty degrees are exact probabilities. Hence, in this context, the use of ordinal conditional functions or possibility theory frameworks also make sense and needs to be explored.

The aim of this paper is to investigate a new extension called product-based possibilitic DL-Lite, denoted by Pb-π-DL-Lite. To the best of our knowledge, there is no work that extends description logics, including lightweight description logics such as DL-Lite to product-based possibility theory.

This paper contains three main contributions. First, it defines the language of Pb-π-DL-Lite and its associated semantics. Then, it provides an encoding of computing the inconsistency degree of a product-based possibilistic DL-Lite knowledge base (KB) (the basis of query answering in Pb-π-DL-Lite) using a weighted maximum 2-Horn satisfiability problem. The converse transformation is also provided. Finally, the paper shows how to use a greedy algorithm, proposed as an approximate method for the weighted set cover problem, to compute an approximate value of the inconsistency degree of a Pb-π-DL-Lite knowledge base. This allows us to have an approximate algorithm for handling instance checking queries as well.

The paper is organized as follows. Section 2 gives a refresh on a DL-Lite language. Section 3 discusses the product-based possibilistic extension of DL-Lite, denoted by Pb-π-DL-Lite, where we present its syntax and its semantics. Section 4 discusses the problem of query answering within Pb-π-DL-Lite. In Section 5, we describe how to compute the inconsistency degree of a Pb-π-DL-Lite knowledge base using Weighted Maximum 2-Horn-SATisfiability (W-Max-2-Horn-SAT) tests. Section 6 is devoted to experimental studies and presents two approximate algorithms: The first one concerns the computation of the inconsistency degree of a Pb-π-DL-Lite knowledge base and the second one, based on the first algorithm, concerns the answering of instance checking queries. Section 7 contains some related work and Section 8 concludes the paper.

2. A refresh on DL-Lite

A description logic language [4] is characterized by a set of constructs used to form complex concepts and roles from atomic ones. It is employed to structure a domain of interest. Each description language allows different sets of constructs. In general, the more expressive is a language, the more expensive (form computational point of view) is its inference process. Hence, one always needs to reach a trade-off between expressiveness and computational issues. DL-Lite (see e.g. [30]) is a good illustration of a language that offers a reasonable expressivity with a tractable query answering [14,13]. DL-Lite is a family of DLs that aims to capture some of the most popular conceptual modeling formalisms. A DL knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ consists of a set $\mathcal{T}$ of concept axioms, role axioms (called TBox) and a set $\mathcal{A}$ of assertional facts (called ABox).

In this paper, we only consider two main members of the DL-Lite family. Namely, the DL-Lite$_{core}$ the core fragment of all DL-Lite logics and DL-Lite$_R$ [14–16]. However, results of this paper can be extended to other fragments of DL-Lite. There are two requirements that should be satisfied: