ARTICLE IN PRESS
IJA:7986

International Journal of Approximate Reasoning ••• (••••) •••–•••

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Influence diagrams for speed profile optimization ☆

Václav Kratochvíl, Jiří Vomlel *

Institute of Information Theory and Automation, Czech Academy of Sciences, Czechia

A R T I C L E I N F O

Article history:
Received 31 March 2016
Received in revised form 11 October 2016
Accepted 25 November 2016
Available online xxxx

Keywords:
Influence diagrams
Optimal control
Vehicle control

A B S T R A C T

Influence diagrams have been applied to diverse decision problems. In this paper, we describe their application to the speed profile optimization problem – a problem traditionally solved by the methods of optimal control theory.

Influence diagrams appeared to be well-suited to these types of problems. It is mainly due to their ability to perform computations efficiently if the utility function is additively decomposed along the vehicle path, which is the case for utility functions based on, e.g., the total driving time or the total fuel consumption. Also, driving constraints can be efficiently included in the influence diagram. If the vehicle speed deviates from the optimal speed profile during the real drive, a new optimal speed profile can be quickly computed in the compiled influence diagram.

The theory of influence diagrams has not yet been sufficiently developed for continuous variables and nonlinear utility functions. We cope with this issue by discretization and by stochastic approximations of deterministic problems.

We performed experiments on a real problem – the speed control of a Formula 1 race car. Influence diagrams can provide a good solution of the problem very quickly. This solution can be used as an initial solution for the methods of the optimal control theory and improves the convergence of these methods.

1. Introduction

Optimization of a vehicle speed profile is a well-known problem studied in the literature. Some authors minimize the energy consumption [16,3,18,4,14,17] while others aim at minimizing the total time [24]. Traditionally, this problem is solved by methods of the optimal control theory [10].

In this paper we describe an application of influence diagrams [7,19,21,9] to the problem of the optimization of a vehicle speed profile, which specifies the recommended vehicle speed at each point on the path. This paper extends our results previously published in two conference papers [11] and [25].

There are two key properties that allow efficient computations with influence diagrams. The first one is that the overall utility function is the sum of local utilities in all considered segments of the vehicle path. This is the case not only when the goal is to minimize the total time, but also when we aim at the minimal total fuel consumption or a linear combination of these two. The second key property is the Markov property. This allows us to aggregate the whole future in one probability and one utility potential. These potentials are defined over the speed variable in the current path segment.

☆ This work was supported by the Czech Science Foundation (project 16-12010S).
* Corresponding author.
E-mail addresses: velorex@utia.cas.cz (V. Kratochvíl), vomlel@utia.cas.cz (J. Vomlel).

http://dx.doi.org/10.1016/j.ijar.2016.11.018

Please cite this article in press as: V. Kratochvíl, J. Vomlel, Influence diagrams for speed profile optimization, Int. J. Approx. Reason. (2017), http://dx.doi.org/10.1016/j.ijar.2016.11.018

We illustrate the proposed method using an example of the speed profile optimization of a Formula 1 race car at the Silverstone F1 circuit [24,26]. The goal is to minimize the total lap time. This example will be used throughout the paper to explain the key concepts and for the final experimental evaluation of the proposed approach. An advantage of this example is that the optimal solution is known [24]. This allows us to compare both the influence diagram solution and the solution found by an optimal control theory method with the analytic solution.

In Section 2 we introduce the physical model of the vehicle using an ordinary differential equation. Since all constraints are naturally given with respect to the vehicle position, we describe the vehicle dynamics with respect to its position. The state variable will be the vehicle speed. In Section 3 we specify the optimal control problem we want to solve, which is to find a vehicle speed profile that minimizes the total time and satisfies all speed constraints.

In Section 5 we describe influence diagrams that can be used for this task. We explain the basic operations with probability and utility potentials. We also discuss the two methods that we use for the inclusion of the speed constraints. In Section 6 we derive an exact optimal algorithm for the continuous influence diagram. It is possible to find the exact solution due to the very specific form of the problem. In a more general setup such a solution would be hard to find. Therefore it is worthwhile to explore the behavior of the discretized version, which we do in Section 7. In this Section we introduce an enhancement used for the values that are on the border of the set of admissible speeds. Since similar problems are often solved using the methods of Nonlinear Optimal Control we briefly discuss the ACADO toolkit, which we use in the experiments to provide us a solution using a common method of optimal control – the multiple shooting method.

The final part of the paper is devoted to numerical experiments. We compare the solutions provided by influence diagrams with different discretizations, and with or without proposed enhancements, with the gold standard, which is the solution provided by the optimal algorithm we propose in Section 6. We also present comparisons with the results of ACADO and with real test pilots at the Silverstone F1 circuit. The experiments show that influence diagrams with the proposed enhancements can be very fast in providing good results that are close to the optimum. Each such solution can be used as a starting point of an optimal control algorithm that can further improve the precision of the solution.

2. The physical model of the vehicle

In this paper, the path of the vehicle will be fixed and known in advance. Therefore it is admissible to define the vehicle position as a distance from the start point. Let $s(t)$ be the vehicle position at time t. The speed at time t is the first derivative of the position with respect to time $v(t) = \dot{s}(t)$ and the acceleration at time t is the first derivative of the speed with respect to time $a(t) = \dot{v}(t) = \ddot{s}(t)$.

All model constraints will be given with respect to the vehicle position; therefore we describe the model dynamics with respect to the position. The state variable will be the speed. By the chain rule for the derivative of a composed function we have

$$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v \quad . \tag{1}$$

From this we get the following ordinary differential equation (ODE):

$$v \cdot \frac{dv}{ds} = a \quad . \tag{2}$$

Let a^{max} be the maximum tangential engine acceleration of the vehicle[1] and a^{min} be the maximum tangential brakes deceleration.[2] Engine acceleration corresponding to position s is defined by the following equation:

$$a^e(u) = \begin{cases} u \cdot a^{max} & \text{if } u > 0 \\ u \cdot a^{min} & \text{otherwise,} \end{cases} \tag{3}$$

where u is a value of the control.[3] Negative values correspond to braking and positive values to accelerating. Let position s be from interval $[0, S]$, where $S \in \mathbb{R}$. The control function u is restricted by

$$-1 \le u(s) \le +1 \quad . \tag{4}$$

Deceleration caused by aerodynamic drag is

$$a^d(s) = c_v \cdot v(s)^2 \quad . \tag{5}$$

The actual vehicle acceleration at position s is

[1] Note that this is a property of the vehicle engine (without considering the aerodynamic drag and friction forces). The real maximum vehicle acceleration is lower.

[2] This is a property of the vehicle brakes (without considering the aerodynamic drag and friction forces). The real maximum deceleration is higher.

[3] We use the letter u because it is standard in control theory.

Please cite this article in press as: V. Kratochvíl, J. Vomlel, Influence diagrams for speed profile optimization, Int. J. Approx. Reason. (2017), http://dx.doi.org/10.1016/j.ijar.2016.11.018