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Uses and computation of imprecise probabilities from statistical data and expert arguments

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ABSTRACT

Imprecise probabilities and the theory of coherent previsions offer a rigorous and powerful framework for modelling subjective uncertainty and solving problems of statistical inference, decision making or risk analysis. The paper introduces formulas for computing imprecise probabilities when statistical data and expert arguments are available to a subject. We then show how to use these imprecise probabilities (dialectical probabilities) for comparing the likelihood of events, conditioning on events, comparing decisions, computing optimal decisions or assessing financial risk. We apply the method to stock trading and show in this experiment the added value both of imprecision and of expert arguments derived from Technical Analysis.

1. Introduction

In forecasting, the term *uncertainty* is used to denote the mental state of a subject who knows in which states a given object of interest can possibly be, but who ignores which state precisely will be the true one at some predetermined time in the future. The possible states are commonly referred to as *scenarios* and the set of all scenarios as *universe*. Uncertainty may be described mathematically in terms of a *Bayesian probability* defined on the universe and quantifying the likelihood or chance that a possible state will be the true one. Broadly speaking, probability has been defined in two ways. *Frequentist probability* [58,67,41] is the interpretation of probability that defines an event's probability as its relative frequency in a large number observations. *Subjective probability* [29,79] interprets probability as the confidence an individual has in the truth of a proposition. It is not necessarily based on any precise computation, but is often a reasonable assessment made by a knowledgeable person or expert.

Nonempty sets of Bayesian probability distributions judged compatible with the information available to the subject are known as *credal sets* [62]. Credal sets have been found appropriate for representing uncertainty in situations where the information is incomplete [100], imprecise [96,101,59], vague [99,28,95] or contradictory [93]. The minimum of an event's probability when the distribution ranges over a credal set is called a *lower probability* and its maximum an *upper probability*. Pairs of lower and upper probability distributions obtained from a common credal set constitute *imprecise probabilities* [91,93,94]. The most notorious examples of imprecise probabilities used in Imprecise Probability, Robust Statistics, Generalised Information Theory or Artificial Intelligence are Choquet capacities of order 2 [25], Choquet capacities of infinite order which are more commonly known as belief functions [82,80], possibility and necessity measures [99,31,28] and epsilon-contamination models (also known as gross-error models or vacuous-linear mixtures) [50,11].

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In this article, we assume given a finite universe and consider that the subject disposes of i) a statistical sample of observations of the past occurrences of these scenarios and ii) the opinions of a group of forecasting experts. By *expert*, we simply mean a person, device or system capable of elaborating *arguments*, i.e. justified claims [60,43]. There can be any number of experts and each expert may formulate any number of arguments. We do not care here about who formulated which argument (mapping from experts to arguments), as long as each argument is a justified claim. Expert *opinions* are thus simply modelled by sets of arguments. For the purpose of forecasting, we restrict our attention to two types of arguments only, namely, forecast and mitigation arguments. We refer to sets of scenarios as *events* and say that an event occurs if once revealed the true but initially unknown scenario belongs to the event. A *forecast argument* is a justified claim supporting the occurrence of an event. Forecast arguments are hints that help us at assessing the likelihood of events. A *mitigation argument* is a justified claim according to which the justification provided for another argument (often advanced by a different expert) is suspicious or erroneous. Mitigation arguments are useful for modelling conflicts between expert opinions and mitigate untenable arguments. In our view, mitigation arguments are necessary to properly assess the degree of acceptability or *strength* of forecast arguments. So forecast and mitigation arguments allow us to model the opinions of forecasting experts and the disagreements existing between them.

The purpose of this article is a) to present a method for computing imprecise probabilities from statistical data and expert arguments and b) to explain how to use these imprecise probabilities in problems of statistical inference, decision making or risk analysis. The article is organised in the following way. Section 2 explains what argumentation is, how abstract argumentation can be used to model expert opinions about the occurrence of events and introduces measures of argument strength. Section 3 introduces imprecise probabilities, the concept of coherent prevision and useful principles for statistical inference, decision making and risk analysis under imprecise probabilities. Section 4 introduces the general framework for combining statistical data and forecast arguments from experts and computing imprecise probabilities called *dialectical probabilities*. Section 5 provides a detailed analysis and interpretation of dialectical probabilities. Section 6 is dedicated to the use of dialectical probabilities for statistical inference, decision making and risk analysis. Section 7 is a practical and detailed application to stock trading, which aims at showing the added value of imprecise probabilities and argumentation. We discuss related work in Section 8 and conclude with Section 9.

2. Background on argumentation theory

2.1. What is argumentation?

Uses of argumentation form a recent branch of logic and artificial intelligence. From a historical point of view [24], modelling arguments appear to be at the foundation of artificial intelligence's understanding of rule-based systems where rules can come into conflict. When a rule supporting a conclusion may be defeated by new information, it is said that such reasoning is defeasible. When we chain defeasible reasons to reach a conclusion, we have *arguments*, instead of proofs. Broadly speaking, arguments can therefore be interpreted as justified claims or objects that have the form of logical proof, but that do not have the force of logical proof [60].

Nonetheless, the concepts of mathematical proof and argument can be defined in the same manner [9], by defining means for expressing assertions, accepted bases on which to build theorems from axioms, procedures by which further theorems may be derived from existing theorems and axioms, and precise concepts of termination for derivation. The concept of argument generalises the one of proof, insofar as arguments can be derived from uncertain information, taking the form of defeasible rules or defeasible beliefs. This characteristic is what gives them their defeasible character. Since assumptions are adopted subjectively, arguments can also be said to be subjective. Therefore, arguments are the basis of opinions and it is necessary to keep in mind that their primary role is to persuade, not to find universal truth.

Research in argumentation covers a rather wide collection of topics. The earliest studies of argumentation are of philosophical nature and concerned with the nature, structure and representation of arguments [97,88]. In the early 90s, argumentation started to be used as a way of representing uncertain knowledge [61], to model common sense [64] and default reasoning [70] as well as reasoning under uncertainty [45]. Abstract argumentation [33] then offered a unified theory of non-monotonic reasoning and logic programming.

Abstract argumentation has since then favoured the development of logical approaches to epistemic reasoning [17,74,13,35,34,86,14,87]. A parallel evolution has been the development of logics of argumentation for reasoning under uncertainty [60,18,68,43] which together with abstract argumentation have strongly stimulated the emergence of argumentation-based methods for practical reasoning [6,2,7,73].

Other important results in the field of argumentation are refinements concerning the notion of argument acceptability [3,4,21,36] and their computation [90,36] or complexity [30,37–39], studies on the persuasiveness and explanatory power of arguments [10,65,5], the design of argument strength measures [53,13,43,22,63], the analysis of strategies of argumentation [40,75], the design and study of expert systems and decision support systems [42,44,54,66] and the automated interpretation and generation of human-like (enthymematic) arguments [103].

2.2. Modelling expert opinions on events by abstract argumentation

We assume given in the rest of the article a finite set of $n \geq 1$ *scenarios* called *universe* $\Omega = \{w_1, \dots, w_n\}$. We assume that the set of scenarios is exhaustive and that the scenarios are mutually exclusive, which means that one of the scenarios

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