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Nondeterministic fuzzy automata with membership values in complete residuated lattices

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ABSTRACT

Automata theory based on complete residuated lattice-valued logic has been initiated by Qiu, and further systematically established by Qiu et al. In this paper, we propose the notion of nondeterministic fuzzy automata with membership values in a complete residuated lattice $\mathcal{L}$, called lattice-valued nondeterministic fuzzy automata ($\mathcal{L}$-NFAs). In our setting, a state of an $\mathcal{L}$-NFA may have more than one transition labeled by the same input symbol, which reflects nondeterminism. To compare the behaviors of $\mathcal{L}$-NFAs, we introduce two language equivalence relations which have different discriminating power. Furthermore, we extend the two relations to the complete residuated lattice-valued setting and investigate their properties such as robustness and compositionality. The theory developed here is applicable to the quantitative modeling and verification of fuzzy systems.

1. Introduction

Automata [4,22,33] are one of the important modeling tools in computer science. They are employed in a number of areas of computer science, such as software engineering, lexical analysis, the description of natural languages, programming languages, and verification tools.

The notion of automata was first fuzzified by Wee [49] in 1967. Since then, fuzzy automata theory has been developed by many researchers. Usually, fuzzy automata take values in the unit interval [0, 1] with max–min composition. To enhance the processing ability of fuzzy automata, the membership grades have been extended to more general algebraic structures. Remarkably, Qiu [41] established the basic framework of automata theory based on complete residuated lattice-valued logic [7,20]. Since then, there are many works on fuzzy finite automata with membership values in a complete residuated lattice [14–16,42,44,50,52]. Automata theory based on lattice-ordered monoids has been established by Li and Pedrycz in [26] and has been systematically studied in [27,30,31]. More recently, Li [29] considered finite automata with membership values in an arbitrary lattice. It is worth noting that fuzzy automata and related models are widely used in modeling fuzzy discrete event systems [9,18,25,43,51].

Although the structures of truth values of these fuzzy automata are different, a common feature is that such automata are deterministic in the sense that the fuzzy transition function of an automaton takes a state and an input symbol as

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arguments and returns a possibility distribution on states. Recently, Cao et al. [12,13] introduced nondeterministic fuzzy finite automata with max–min composition and truth values in [0, 1] to handle fuzziness and nondeterminism in system modeling. The fuzzy transition function of a nondeterministic fuzzy finite automaton takes a state and an input symbol as arguments and returns one, or more possibility distributions on states.

In this paper, we extend the models in [12,13] to a more general framework taking the truth values in a complete residuated lattice $\mathcal{L}$. Such a model is referred as lattice-valued nondeterministic fuzzy automata ($\mathcal{L}$-NFAs). Our framework subsumes some well-known fuzzy systems such as fuzzy automata taking truth values in complete residuated lattices. There are two main reasons that we choose complete residuated lattices as the structure of truth values of $\mathcal{L}$-NFAs. One is that complete residuated lattices are a general algebraic structure with very important applications [7,20,34–36,38–40,45], and the other is that the richness of the structure allows us to establish quantitative verification relations between $\mathcal{L}$-NFAs.

To describe the behavior of $\mathcal{L}$-NFAs, we need to separate nondeterminism from fuzziness, which is achieved by means of strategy. Our strategy, which is similar to [3,37,46], chooses a next transition to schedule, based on the past history and the input symbol of current state. When a strategy is applied to a system, a fully fuzzy model called a resolution is obtained. To define the fuzzy languages accepted by an automaton, we first define the fuzzy languages accepted by all its resolutions, and then lift them to the whole system. By the way, we show that our approach to defining fuzzy languages coincides with the one in [12,13].

A classical notion on fuzzy automata is language equivalence, which says that two fuzzy automata are equivalent if they accept the same fuzzy language. Using this notion, Li et al. [26] provided some systematic results about the relationships among several important types of fuzzy automata. Moreover, Cao et al. [12] showed that fuzzy automata, nondeterministic fuzzy automata, and nondeterministic fuzzy automata with ε-moves are equivalent in the sense that they accept the same class of fuzzy languages.

We need to extend the concept of language equivalences for $\mathcal{L}$-NFAs. To this end, we first define a weak language equivalence on $\mathcal{L}$-NFAs. One main result on this relation is that the expressive powers of $\mathcal{L}$-NFAs and fuzzy automata with membership values in complete residuated lattices are the same from the point of view of weak language equivalence. We then define another language equivalence on $\mathcal{L}$-NFAs, called strong language equivalence. It says that two $\mathcal{L}$-NFAs are strongly language-equivalent if for each word α and for each resolution of one $\mathcal{L}$-NFA, there exists a resolution of the other $\mathcal{L}$-NFA such that the truth values of the word accepted by them are the same. The first relation is inspired by the work on nondeterministic fuzzy automata [12,13], while the latter is motivated by the work on nondeterministic and probabilistic labeled transition systems [5,6]. Contrary to weak language equivalence, the expressive power of $\mathcal{L}$-NFAs is more powerful than that of fuzzy automata from the point of view of strong language equivalence, and moreover, we show that strong language equivalence is strictly finer than weak language equivalence.

Note also that the two relations are all boolean notions: Given two $\mathcal{L}$-NFAs, each relation returns a boolean answer indicating whether or not the two automata are equivalent. As emphasized in [1,2,21,34–36,47,54–56], boolean equivalence verification relation, which divides a system into equivalent or inequivalent parts, falls short of the practical need to assess the behavior of systems in a more nuanced fashion against multiple criteria. In light of this, we extend each language relation to the complete residuated lattice-valued setting such that giving any two automata returns a truth value in the complete residuated lattice which measures the degree of behavioral equivalence between the automata. After discussing the relationship between the two fuzzy relations, we show that the two fuzzy relations have a number of properties which are necessary for quantitative verification: (1) The two fuzzy relations are both fuzzy equivalence relations, which means that such relations can be applied to implementation verification. (2) They are both compositional for product composition operator. This allows us to use the two relations for modular reasoning. (3) In some practical applications, the target set of a transition of an $\mathcal{L}$-NFA may be somewhat imprecise and subjective, since it comes from experience or theoretical estimations. It inspires us to investigate the robustness of the two relations. We consider it for fuzzy automata with membership values in finite Heyting algebras, an important fragment of complete residuated lattices.

The idea of extending fuzzy language equivalence and/or inclusion relations between fuzzy automata to the fuzzy setting was first studied by Kupferman et al. in [23], where they defined the implication value as an extension of language inclusion with membership values in finite De Morgan lattices and showed that the problem of computing the implication value is PSPACE-complete. The fuzzy version of our Definition 4.1 (see below) and similar notions in [24,28,32] have been used to minimize fuzzy finite automata over complete residuated lattices. For example, employing the concept of an approximate equivalence of fuzzy automata, Bělohlávek and Krupka [8] gave a procedure of constructing a crisp-deterministic fuzzy automaton whose fuzzy language is approximately equal to the fuzzy language of the given one. It is worth noting that the existing results on fuzzy automata, except for [11], are applicable only to finite (i.e., finite-state and finite-alphabet) fuzzy systems. Clearly, it is a limitation, because many fuzzy systems are infinite-state or infinite-alphabet. For example, fuzzy hybrid machines in [19] are infinite-state and infinite-alphabet and fuzzy automata for computing with all words in [10,48,53] require an infinite alphabet as well. The results obtained in the paper are applicable to both finite and infinite systems.

Structure of the paper The rest of the paper is organized as follows. After reviewing some basic facts on complete residuated lattices and fuzzy sets in Section 2, we propose the notion of nondeterministic fuzzy automata and related concepts in Section 3. We introduce the two language equivalence relations to compare the behaviors of $\mathcal{L}$-NFAs and extend the two relations to the complete residuated lattice-valued setting in Section 4. We discuss some properties of the two fuzzy relations in Section 5, and conclude the paper in Section 6.