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Robust identification of highly persistent interest rate regimes ☆

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A B S T R A C T

Parametric specifications in State Space Models (SSMs) are a source of bias in case of mismatch between modeling assumptions and reality. We propose a Bayesian semiparametric SSM that is robust to misspecified emission distributions. The Markovian nature of the latent stochastic process creates a temporal dependence and links the random probability distributions of the observations in a mixture of products of Dirichlet processes (MPDP). The model is shown to be adequate and it is applied to simulated data and to the motivating empirical problem of regime shifts in interest rates with latent state persistence.

1. Introduction

Many well known models, such as ARMA, ARIMA, Structural Time Series Models, Unobserved Components Models, Dynamic Regressions and Spline-Fitting Models, can be reformulated as State Space Models (SSM). A SSM requires the specification of the functional dependence over time of the latent stochastic process, the functional dependence between the observation and the latent variable, the distributions of the measurement and of the transition errors. Each specification is a potential source of bias when the true data generating process is different from the dynamics implied by the model, emphasizing the need for robust methodologies.

A Bayesian semiparametric approach is a possible solution that has received increasing attention in recent literature. Other possibilities are: Robust filters for uncertain parameters, demonstrated by the Riccati approach of Xie et al. [48] and Xie et al. [49] and the Linear Matrix Inequality approach used by Li and Fu [36], but both these alternatives deal with the problem of parameter uncertainty but do not manage functional uncertainty. Ex-post diagnostic checking on standardized residuals is another possibility [33], but this technique only provides a diagnostic test without giving an indication of which model to use.

Various approaches have been proposed in the literature that extend the SSM to a Bayesian semiparametric framework. Ghosh et al. [29] introduce SSMs with unknown measurement and transition equations, using Gaussian processes as priors.

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Their method makes the SSM more robust, regarding the functional dependencies in the two equations. Caron et al. [6] use Kalman filtering techniques to estimate SSMs with noise probability density functions of unknown form. They propose a Bayesian nonparametric noise model based on Dirichlet Process Mixtures (DPM) with Gaussian kernels, estimated through MCMC and Sequential Monte Carlo methods. Beal et al. [3] start with the assumption that the data is generated by a discrete state variable and formulate Hidden Markov Models (HMM) with a countably infinite number of hidden states. By using the theory of Dirichlet Processes (DP, [23,24]) Beal et al. implicitly integrate out the infinitely many transition parameters, leaving three hyper-parameters to be learned from the data. Teh et al. [46] formalizes Beal et al.'s work in a Bayesian framework, proposing a HMM which allows for an unbounded countable set of states through a set of DPs, one for each value of the current state. Caron et al. [6], Beal et al. [3] and Teh et al. [46] address the issue of uncertainty in the transition error distribution, with Beal et al. [3] and Teh et al. [46] assuming a time-discrete and space-discrete SSM.

We consider a Bayesian semiparametric approach for SSMs, robust to the case of a misspecified emission distribution and able to handle possibly persistent latent states. Our motivating application is the robust identification of highly persistent regime shifts in interest rates. Latent states in the economy affect both the mean and the variance of interest rates, which characterize interest rate regimes. These latent states tend to be highly persistent, with some states well represented in the sample, whilst others only appear in few observations. A model based solely on the empirical distributions would be unreliable for infrequent economic states, yet a parametric model can be too restrictive for well represented states. The intermediate nonparametric approach that we consider has the benefit of combining both the information contained in the empirical distributions with parametric model assumptions.

Interest rates are clustered in latent classes and within each class observations are assumed to be exchangeable. Each observation is assigned to one of the latent classes, according to the distinct value of a latent stochastic process (that is, the latent state of the economy). We can then build a parsimonious and robust framework that accommodates the arbitrary forms of the response distribution, assuming that the response distribution function, given each latent state of the economy is unknown and sampled from a DP. The latent states present a Markovian dependence that links the random distributions of different latent states.

In the proposed model, the base distribution of the DP is the emission distribution of a corresponding parametric SSM, and it depends on the value of the latent stochastic Markov process. The DP concentration parameter is also state-dependent and can be learned from the data, so that the extent to which predictions and estimates deviate from the mean parametric SSM, depends on the observations: latent states which occur more frequently give more weight to the data, whilst, for infrequent states, the inferential conclusions will be more anchored to the base parametric model. Finally, the simple parametric dependence structure of the latent stochastic process permits a robust modeling framework that properly accounts for persistence in the latent states. This last property is essential in our motivating empirical example: the two latent states of the economy are inferred to persist in their states with probability of, respectively, 92% and 76%. The ability of our model to capture state persistence renders it suitable to our financial problem. The Bayesian semiparametric model of Teh et al. [46] is less adequate in applications which show strong state persistence [27,28,38] and, differently from our approach which focuses on the emission distribution, it focuses on making the transition equation specification more robust.

In Section 2 we present our model, and highlight differences with alternative approaches available in the literature. Posterior and predictive analysis is conducted in Section 3, and the theoretical justification of the proposed model is given in Appendix. In Section 4 we test our procedure on a simulated Hidden Markov Model. The motivating financial application is studied in Section 5: a Bayesian semiparametric Markov switching model for interest rates. These applications show that it is possible to extend parametric SSMs to Bayesian semiparametric SSMs, without adding heavy computational burdens and with considerable gains in terms of posterior and predictive performance. Section 6 contains concluding remarks.

2. Model construction and properties

Consider $y = \{y_1, y_2, \ldots, y_T\}$, time observations such that, given F_t and θ_t for all $t = 1, \ldots, T$, $y \sim \prod_{t=1}^T F_t(y_t|\theta_t)$. Let k be the number of distinct values θ_i^* in $\theta = \{\theta_1, \theta_2, \ldots, \theta_T\}$. We do not distinguish the notation for probability measures and distribution functions. Define $\theta^* = (\theta_1^*, \ldots, \theta_k^*)$, for $k \le T$. Also define, for $i = 1, \ldots, k$,

$$C_i = \{t \in (1, \ldots, T) : \theta_t = \theta_i^*\},$$

with cardinality n_i. Then, conditional on θ and $F_1, \ldots, F_k$, the joint distribution of y is

$$\mathbb{P}(y_1 \le y_1, \ldots, y_T \le y_T | F_1, \ldots, F_k, \theta) = \prod_{t \in C_1} F_1(y_t|\theta_1^*) \prod_{t \in C_2} F_2(y_t|\theta_2^*) \cdots \prod_{t \in C_k} F_k(y_t|\theta_k^*).$$

This means that the elements of y are independent, conditionally on the knowledge of θ and $F_1, \ldots, F_k$. Furthermore, $F_1, \ldots, F_k$ are not known distribution functions, but are uncertain and selected from the Mixture of Products of Dirichlet Processes (MPDP) introduced by Cifarelli and Regazzini [13]. More precisely, given the vector θ, we assume that F_i is sampled from a Dirichlet Process [23,24] $\mathcal{D}$, with parameter $\alpha H_{\theta_i^*}$, where α is the concentration parameter and $H_{\theta_i^*}$ the