International Journal of Approximate Reasoning ••• (••••) •••–•••

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Exploiting collective knowledge with three-way decision theory: Cases from the questionnaire-based research ☆

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A R T I C L E I N F O

Article history:
Received 8 July 2016
Received in revised form 7 October 2016
Accepted 16 November 2016
Available online xxxx

Keywords:
Three-way decision
User study
Online questionnaires
Collective knowledge

A B S T R A C T

Two methods are proposed for collective knowledge extraction from questionnaires with ordinal scales and dichotomous questions.
Both methods are based on a three-way decision procedure and a statistical method aimed at attaining statistical significance of the above decision. One method is aimed at giving an (absolute) assessment of "objects" according to a given "criterion" and the other one at producing a relative ranking of the "objects". A criterion can be related to one or more questionnaire items (usually questions or statements). In this latter case a method to compose ordinal items in aggregate scores is also given. The paper also presents two various case studies that illustrate the methods and give motivations for their application in different domains where the knowledge of a community or any distributed group of experts can be externalized (in terms of users' perceptions, attitudes, opinions, choices) with a structured closed-ended questionnaire.

1. Introduction

Three-way decision (3WD) theory is based on a trisecting-and-acting paradigm and it aims to classify a finite collection of objects in three regions (positive, negative, boundary) in order to act differently on objects belonging to different regions [1, 2]. It was introduced in 2009 by Yiyu Yao [3] and despite the fact that it is a young research field, it is gaining a consensus in the rough-set community and beyond [4] leading towards a standalone discipline. Due to its relative novelty, a number of issues are still open and Yao in [2] outlines some directions for future works. With respect to trisection of the universe, which is the central point of the methodology, open problems include

> to consider a ranking of objects, which is a fundamental notion in theories of decision-making. Three regions correspond to the top, middle, and bottom segments of the ranking.

Moreover, a further research direction Yao identified was to "seek for a statistical interpretation of a trisection". First steps to this aim can indeed be found in [5], where basic statistical measures such as average and standard deviation are used to partition the universe.

☆ This paper is part of the virtual special issue on tri-partition, edited by Davide Ciucci and Yiyu Yao.

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http://dx.doi.org/10.1016/j.ijar.2016.11.013

Here, we are going to delve into the two research concerns mentioned above in an integrated manner, namely, to provide a three-level ranking of objects using a statistical method and interpretation. The general criterion that we propose to divide a universe U into three pair-wise disjoint regions is the *collective perception*[1] of a population of domain experts.

This requires to ask n experts (i.e., a sample from a population) if an object x (e.g., a scientific conference, a diagnosis, a medical treatment) on the basis of some criteria (e.g., the quality of the conference x, the plausibility of diagnosis x, the appropriateness of therapy x) should be classified, that is put, in a specific partition of U (e.g., class of quality, level of plausibility, strength of appropriateness).

Then, a simplistic way would be to ask each expert to put object x in a partition (or to abstain to move it from the border) and then to adopt the majority criterion: the object x is moved to a partition if the majority of the sample decided to put it there. However this procedure exhibits two main shortcomings. First, it is subject to what we could call "dichotomous choice" bias, that is the bias coming from the fact that people have to decide between two alternatives but they are not really convinced and take the decision almost by coin flipping or, worse yet, subject to some *cognitive effect* (like *priming* [7]). In this case, even a small effect can be amplified by the number of subjects involved. Second, the partition would express the collective decision (deliberation) of a specific group of people (the sample), but nothing could be said of the overall population. A priori, a different sample could bring to a completely different partitioning.

To go beyond this latter shortcomings (probably the worst one) an alternative approach would be to estimate whether the population, from which the sample has been extracted, expresses any majority-ruled consensus. To this aim, the real proportions of both two kinds of responses (that is "x should be put in U_i" or it should not) can be estimated from the proportions extracted from the sample of respondents by performing a hypothesis test (e.g., the chi-squared test).

In this paper, we adopt a similar approach of statistical inference, while also addressing the dichotomous choice bias. To this latter aim, we do not ask directly the experts in which partition of U they would put x; rather, for each respondent we infer her/his classification by first analyzing her/his absolute evaluations of x recorded on a psychometric ordinal scale (e.g., a Likert scale [8,9]). This, if the scale is sufficiently wide, could give the respondents a way to express either a strong attitude and convinced resolution in either directions (one partition or the latter) by choosing the outer or extreme values of the scale or, conversely, express a weaker conviction by choosing the middle values of the scale. And then by considering the aggregate classification through a statistical procedure. This latter one is aimed at rejecting the *null hypothesis* that no partitioning decision can be made beyond the effects due to chance (that is for biases due to the specific sample under consideration, in the assumption that it has been randomly extracted from the reference population and it is representative of the whole population for the characteristics of interest, e.g., expertise).

More specifically the statistical hypothesis test would provide a number to be compared with an acceptance threshold and hence it would give an indication on whether the perceptions collected from the sample of respondents allow us to reject the null hypothesis above (assumed as true), or not. Thus, the result of the hypothesis test represents the so-called *evaluation function* of the three-way method [2]. The reader should note that in our approach there is not a "right" classification, so we cannot compute the thresholds by minimization/maximization of a quantity measure, as it is the case in other applications of three-way decision theory. We rather aim at the *most representative* classification of a community of experts, which is inherently 3-partitioned for the difficulty of reaching a large consensus especially in non-trivial matters.

As a first step of our method, we thus produce what we could call an *absolute* classification of objects in three regions. We remark that this classification is obtained through two main resources: the *collective knowledge* [10] that is available in a community of experts, and a *statistical procedure* through which to tap in this knowledge [11].

Further, also a *relative* classification in terms of a ranking of objects is obtained, again by a three-way decision process. In this case, the three regions of U are those related to the capability to put an object into any of the "first positions" (e.g., the first three positions, like in a podium), or in any of "the other positions" *beyond any doubt* (that is beyond the effect on the above decision that is due to chance), or to the statistical incapability to do so (because the user responses do not allow to reject the hypothesis stating that the object could not put in either of the above partitions). Also in this case we propose to use ordinal values (i.e., categories from a total order set), but indirectly: by conjecturing that respondents are better in giving absolute judgments than rankings, especially when the entities to rank are many and hence differences among them subtle (if any), we derive a ranking for each respondent, and then we look for any tendency in the ranking distribution beyond the effects due to chance.

The paper is structured as follows. In Section 2, we give an overall picture of our approach and explain the method to compose ordinal variables together. Section 3 contains the two main methods of our contribution: the three-way assessment and ranking procedures based on questionnaire (ordinal) responses. These two methods are then applied to two case studies in Section 4: the quality evaluation of scientific conferences and the management of difficult medical cases, which we chose for their heterogeneity and controversial or difficult practices of decision making, respectively. Finally, in Section 5 we present the conclusions and outlines for future research.

[1] "A perception is defined as the result of a cognitive process whereby a person interprets information" [6]. A collective perception is then the perception that can be inferred from a collective of people, each of whom expresses an individual perception. To our aims such an abstraction can be expressed in terms of a stochastic variable.

Please cite this article in press as: F. Cabitza et al., Exploiting collective knowledge with three-way decision theory: Cases from the questionnaire-based research, Int. J. Approx. Reason. (2016), http://dx.doi.org/10.1016/j.ijar.2016.11.013