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Model validation and stochastic stability of a hydro-turbine governing system under hydraulic excitations

Beibei Xu [a], Diyi Chen [a,*], Silvia Tolo [b], Edoardo Patelli [b], Yanlong Jiang [c]

[a] Institute of Water Resources and Hydropower Research, Northwest A&F University, Yangling, Shaanxi 712100, PR China
[b] Institute for Risk and Uncertainty, Chadwick Building, University of Liverpool, Peach Street, Liverpool L69 7ZF, United Kingdom
[c] The Yellow River Qinghai Hydropower Development Co., Ltd., Xining, Qinghai 810000, PR China

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ABSTRACT

This paper addresses the stability of a hydro-turbine governing system under hydraulic excitations. During the operation of a hydro-turbine, water hammer with different intensities occurs frequently, resulting in the stochastic change of the cross-sectional area (A) of the penstock. In this study, we first introduce a stochastic variable u to the cross-sectional area (A) of the penstock related to the intensity of water hammer. Using the Chebyshev polynomial approximation, the stochastic hydro-turbine governing model is simplified to its equivalent deterministic model, by which the dynamic characteristics of the stochastic hydro-turbine governing system can be obtained from numerical experiments. From comparisons based on an operational hydropower station, we verify that the stochastic model is suitable for describing the dynamic behaviors of the hydro-turbine governing system in full-scale applications. We also analyze the change laws of the dynamic variables under increasing stochastic intensity. Moreover, the differential coefficient with different values is used to study the stability of the system, and stability of the hydro-turbine flow with the increasing load disturbance is also presented. Finally, all of the above numerical results supply some basis for modeling efficiently the operation of large hydropower stations.

1. Introduction

By 2014, the total installed capacity of sustainable water energy in China, representing approximately 25% of the worldwide installed capacity, exceeded 3×10^8 kW. Furthermore, the installed global hydropower capacity is expected to double in the next 30 years [1–3], bringing it to 2×10^9 kW. Clearly, hydropower has a promising prospect. Several challenging problems, however, exist in the operation of large hydropower stations. These problems include the vibrations of hydro-turbine generator units, the occurrence of water hammer in the penstock, and the increasing randomness of electric loads due to diverse power generation sources [4–6]. These problems are inseparably linked with the regulation of hydro-turbine governing systems. In recent years, studies of the hydro-turbine governing system have been mainly divided into two categories. The first category focuses on operational conditions and the hydro-structure of hydropower stations [7–13]. The second category focuses on the mathematical models of hydropower stations to optimize dynamic behaviors in terms of hydro-turbine control [14–22]. Conversely, the effects of the water hammer on the penstock are rarely considered in the mathematical modeling of hydro-turbine governing systems.

Water hammer is a commonly recognized general problem in transmission penstocks, and occurs when there is an abrupt change of flow in the penstock. Some possible causes leading to water hammer include, among others, the startup (or shutdown) of hydro-turbine generator units, rapid change in transmission conditions, and opening and closure of valves [23–27]. Moreover, high-intensity water hammer can lead to significant damages and even disruption of the hydro-turbine governing system [28–31]. The propagation process of water hammer occurring in the penstock can be divided into four stages, i.e. the compression process, the recovery process, the expansion process, and another recovery process. During the propagation process, frequent flow changing in the penstock makes water hammer with different intensities arise continuously, which leads to the stochastic change of the cross-sectional area A of the penstock.

In light of these considerations, four significant innovations are presented in this paper. First, for a large hydropower station, we propose a stochastic model of the hydro-turbine governing system. Moreover, as pioneering research, we reduce the stochastic model to its equivalent deterministic model by using the Chebyshev poly-

* Corresponding author at: Institute of Water Resources and Hydropower Research, Northwest A&F University, Yangling, Shaanxi 712100, China.
E-mail address: diyichen@nwsuaf.edu.cn (D. Chen).

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nomial approximation. Second, from numerical experiments based on a large currently operating hydropower station, we verify that the stochastic model is suitable for describing the behaviors of the hydro-turbine governing system in the operational process. Third, the effect of the stochastic intensity D on the stability of the above system is analyzed. Fourth, we present the laws of stable ranges of the hydro-turbine flow q, the guide vane opening y, and the head loss h_q at the hydro-turbine entrance under different conditions.

The rest of this paper is organized as follows. Section 2 presents the modeling process of the hydro-turbine governing system. In Section 3, the stochastic model of the hydro-turbine governing system and its simplified deterministic model are proposed. Numerical experiments along with detailed analyses are presented in Section 4. Finally, Section 5 summarizes the results.

2. Mathematical modeling of a hydro-turbine governing system

From Newton's second law of motion, the dynamic mathematical equations of the penstock system are

$$\begin{cases} h_t = h_r - h_f \\ h_f = f_1 q^2 \\ h_q = Z_{01} q \tanh(T_{01} s) \end{cases} \tag{1}$$

where h_r is the relative value of the rated head, h_t is the relative value of the hydro-turbine head, h_f is the friction head loss in the penstock, h_q is the head loss at the hydro-turbine entrance, and f_1 is the friction factor of the penstock. The head loss h_q, considering the elastic water hammer effect, can be written as [4,19]

$$h_{q1}(s) = Z_{01} \frac{\pi^2 T_{01} s + T_{01}^3 s^3}{\pi^2 + 4T_{01}^2 s^2} q_1 \tag{2}$$

Turning Eq. (2) into the state-space equations results in

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -\frac{\pi^2}{T_{01}^2} x_2 + \frac{1}{Z_{01} T_{01}^3} h_{q1} \\ \dot{q} = -3\pi^2 x_2 + \frac{4}{Z_{01} T_{01}} h_{q1} \end{cases} \tag{3}$$

where T_{01} is the elastic time constant of the penstock system, $T_{01} = \frac{L}{\upsilon}$; L is the length of the penstock; υ is the speed of the surge pressure wave in the penstock; and Z_{01} is the resistance value of the hydraulic surge in the penstock system, which can be expressed as

$$Z_{01} = \frac{\upsilon Q_r}{A g H_r} \tag{4}$$

where Q_r is the rated flow, H_r is the rated head, g is the acceleration of gravity, A is the cross-sectional area of the penstock, and q is the relative value of the hydro-turbine flow.

For a synchronous generator system, a first-order mathematical model is used, which is

$$\dot{\omega} = \frac{1}{T_{ab}} (m_t - m_{g0} - e_n \omega) \tag{5}$$

where ω is the angular speed of the generator, ω_0 is the rated angular speed of the generator, e_n is the accommodation coefficient, m_{g0} is the load disturbance of the generator, and m_t is the output torque of the hydro-turbine. The traditional mathematical equation of the output torque for a hydro-turbine, proposed by an IEEE Working Group in 1993, is often adopted in the mathematical modeling of a hydro-turbine governing system [28], which is

$$P_{m-IEEE} = A_t h_t (q - q_{nl}) - D_t y \omega \tag{6}$$

Since the organization structures (the mounting height of the guide vane, the flow angle of the guide vane, etc.) for different types of hydro-turbines are very different, Eq. (6) is just a general equation that cannot reflect the fine characteristics of the output power for a specific hydro-turbine in a transient process. In this paper, the output torque derived using the internal characteristics method is described as [19]

$$\begin{cases} h_t(t) = \frac{\omega}{g} \left[\left(\frac{\cot\gamma}{2\pi b_0} + r \frac{\cot\beta}{F} \right) q(t) - \omega r^2 \right] \\ m_t(t) = \rho \left[\left(\frac{\cot\gamma}{2\pi b_0} + r \frac{\cot\beta}{F} \right) q_1 - \omega r^2 \right] \end{cases} \tag{7}$$

where γ is the flow angle of the guide vane, β is the flow angle of the middle area of the runner, b_0 is the mounting height of the guide vane, r is the radius of the middle area of the runner, and F is the area of the exit of the runner.

The hydraulic servo model is

$$T_y \frac{dy}{dt} + y - y_0 = u \tag{8}$$

where T_y is the major relay connecter response time of the hydraulic servo model; y_0 is the initial incremental deviation of the guide vane opening; and u is the output signal of the hydraulic servo model, which is described by Eq. (9):

$$u = k_p(r - \omega) + k_i \int (r - \omega) dt + k_d (r - \dot{\omega}) \tag{9}$$

From Eqs. (1)–(9), the dynamic mathematical equations of the hydro-turbine governing system are

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = -\frac{\pi^2}{T_{01}^2} x_2 + \frac{1}{Z_{01} T_{01}^3} (h_0 - f q^2 - h_t) \\ \dot{q} = -3\pi^2 x_2 + \frac{4}{Z_{01} T_{01}} (h_0 - f q^2 - h_t) \\ \dot{\omega} = \frac{1}{T_{ab}} (m_t - m_{g0} - e_n \omega) \\ \dot{y} = \frac{1}{T_y} (k_p (r - \omega) + k_i x_4 - k_d \dot{\omega} - y + y_0) \\ \dot{x}_4 = r - \omega \end{cases} \tag{10}$$

Considering $c = \frac{4}{Z_{01} T_{01}}$, $a = -\frac{\pi^2}{T_{01}^2}$, and $b = \frac{1}{4T_{01}^2} c$, Eq. (10) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = a x_2 + \frac{1}{4T_{01}^2} c (h_0 - f q^2 - h_t) \\ \dot{q} = -3\pi^2 x_2 + c (h_0 - f q^2 - h_t) \\ \dot{\omega} = \frac{1}{T_{ab}} (m_t - m_{g0} - e_n \omega) \\ \dot{y} = \frac{1}{T_y} (k_p (r - \omega) + k_i x_4 - k_d \dot{\omega} - y + y_0) \\ \dot{x}_4 = r - \omega \end{cases} \tag{11}$$

3. Mathematical modeling of the stochastic hydro-turbine governing system

Water hammer, which is basically a pressure wave, occurs when there is an abrupt change of flow in the penstock. Due to the effect of the viscoelastic characteristics of the penstock wall, the cross-sectional area of the penstock changes correspondingly during water hammer propagation. Fig. 1 illustrates the change law for the four stages of the water hammer propagation process.

Fig. 1 shows the water in the penstock flowing from position n to position m with a steady flow υ_0 during the stable operation of a hydropower station. As shown in Fig. 1(a), when the water gate at