Electrical Power and Energy Systems 95 (2018) 166–176

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Theoretical convergence guarantees versus numerical convergence behavior of the holomorphically embedded power flow method

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ARTICLE INFO

Article history:
Received 15 June 2017
Received in revised form 24 July 2017
Accepted 21 August 2017

ABSTRACT

The holomorphic embedding load flow method (HELM) is an application for solving the power-flow problem based on a novel method developed by Dr. Trias. The advantage of the method is that it comes with a theoretical guarantee of convergence to the high-voltage (operable) solution, if it exists, provided the equations are suitably framed. While theoretical convergence is guaranteed by Stahl's theorem, numerical convergence is not; it depends on the analytic continuation algorithm chosen. Since the holomorphic embedding method (HEM) has begun to find a broader range of applications (it has been applied to nonlinear structure-preserving network reduction, weak node identification and saddle-node bifurcation point determination), examining which algorithms provide the best numerical convergence properties, which do not, why some work and not others, and what can be done to improve these methods, has become important. The numerical Achilles heel of HEM is the calculation of the Padé approximant, which is needed to provide both the theoretical convergence guarantee and accelerated numerical convergence. In the past, only two ways of obtaining Padé approximants applied to the power series resulting from power-system-type problems have been discussed in detail: the matrix method and the Viskovatov method. This paper explores several methods of accelerating the convergence of these power series and/or providing analytic continuation and distinguishes between those that are backed by the theoretical convergence guarantee of Stahl's theorem (i.e., those computing Pade approximants), and those that are not. For methods that are consistent with Stahl's theoretical convergence guarantee, we identify which methods are computationally less expensive, which have better numerical performance and what remedies exist when these methods fail to converge numerically.

1. Introduction

The nonlinear equations that characterize the power-flow problem are the bedrock equations for many power system analysis and simulation problems. Of the various numerical methods researched in the past to solve the power-flow problem, the Newton-Raphson (NR) method and its variants such as the Fast Decoupled Load Flow (FDLF) algorithms are most widely used in the power industry [1–5]. While these methods have been shown to be very reliable and robust for most problems, they do have convergence issues for ill-conditioned systems. In such cases, if a solution is not found by the NR-based methods, it is not possible to know whether the given problem does or does not have an operable solution or whether the algorithm simply failed to find one. In some rare cases (especially when the operating point is close to the saddle node bifurcation point (SNBP) of the system,) NR can converge to a low-voltage (inoperable) solution. A novel non-iterative (recursive) algorithm, the holomorphically embedded load-flow method (HELM) was proposed by Dr. Antonio Trias which is theoretically guaranteed to converge to the high-voltage (operable) solution if one exists, provided the conditions of Stahl's theorem are satisfied [6–9] and the precision of the computing engine is sufficient.

From its introduction to the power system community in 2012, the holomorphic embedding method (HEM) has been applied to an ever expanding list of applications. The holomorphically embedded power flow formulation (HEPF) was first applied to the power-flow problem with only PQ buses [10]. A model for PV buses was proposed in [11] along with an algorithm which considered the discrete changes in the system such as bus-type switching and tap-changing transformers. The algorithm was extended to dc systems in [12] where it was shown that the method could accommodate the nonlinearities that characterize the I-V curves of power electronics devices. The HEPF theory, adapted for ZIP-load models, was first presented in [14], such that the solution obtained at different values of embedding parameter, α, represented the bus voltages when the loads were scaled by α, and the smallest real pole or

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http://dx.doi.org/10.1016/j.ijepes.2017.08.018
0142-0615/

zero of the Padé approximants was shown to give the load-scaling value corresponding to the SNBP. Such a scalable formulation has also been used to develop network equivalents that remain accurate even as the operating conditions change in [15,40]. A formulation aimed at finding the low-voltage solutions of a power system was proposed in [13] using the HEPF. With the exception of [12], all the above-mentioned manuscripts report using either the matrix method or the Viskovatov method to calculate the Padé rational-function approximants to obtain converged values of the bus voltage series. These methods have the advantage that they provide the voltage solution as a ratio of two (analytical) polynomials in the embedding parameter α, and the analytical nature of this solution expression was used to advantage in [14,15] to estimate the SNBP and critical to generating nonlinear reduced-order equivalent networks [15,40] and weak-bus determination [41]. Knowledge that the poles of the close-to-diagonal Padé approximants accumulate on Stahl's compact set [7,8] was critical to efficient SNBP estimation. A discussion about the various caveats of the Padé approximants such as the defects (also known as Froissart doublets) is provided in [12]. These defects are spurious pole-zero pairs which are transient in nature, i.e., they appear and disappear from an approximant of one degree to another and are not indicators of the true singularities of the function [17], which must be taken into account when estimating the SNBP. Since the power-flow equations are algebraic equations, Stahl's theorem [8] shows that Froissart doublets can occur only due to numerical round-off and not with exact arithmetic. Thus for the power-flow problem, spurious pole-zero pairs can be avoided by increasing the Padé order [12] and using the behavior of the smallest roots of successive approximants to identify these anomalous pairs.

In the remainder of this paper, the holomorphic embedding method applied to the power-flow problem will be referred to as HEPF, in order to distinguish it from HELM which is a patented application whose implementation details are not available in the public domain. The essence of the HEPF method is to convert the original non-holomorphic power-balance equations (non-holomorphic in the voltage variable) into holomorphic functions by suitably embedding a complex parameter α. The voltages are then obtained as a Maclaurin series of α and Stahl's theorem guarantees theoretical convergence provided analytic continuation, via diagonal or near-diagonal Padé approximants, is used to obtain the converged values of the series.

While many articles have been published on HEPF/HELM, these articles discuss only the theoretical convergence guarantee, remaining silent on numerical issues, leaving the reader susceptible to inferring erroneously that the theoretical convergence guarantee necessarily implies a numerical convergence guarantee. Therefore, it is important within the context of this paper to understand what Stahl's theorem does and does not say. First it provides a theoretical convergence guarantee provided rather mild conditions on the nonlinear algebraic equations are satisfied, conditions that the power-flow equations can be structured to satisfy. Theoretical convergence is guaranteed provided diagonal or near-diagonal Padé approximants are used to perform the analytic continuation. Stahl's theorem does not guarantee theoretical convergence with either limited precision or when a limited number of series terms is used. Further Stahl's theorem is silent about whether other (non-Padé-approximant) convergence acceleration and analytic continuation techniques might also guarantee theoretical convergence. The objective of this work is to show that, indeed, while theoretical convergence may be guaranteed when using Padé approximants, numerical convergence is not guaranteed and is dependent on the numerical method chosen. A second objective is to assess whether other convergence acceleration techniques, which may or may not be the silent beneficiaries of an undiscovered theoretical convergence guarantee, have acceptable performance when applied to the power-flow problem.

As an example of the lack of numerical convergence guarantee, we have observed that for heavily loaded systems that are close to SNBP, the HEPF needs more terms and higher precision in order to obtain convergence when the matrix method of calculating Padé approximants is used [16]. This is to be expected since beyond the SNBP, the non-existence of a solution is indicated by the oscillation of the sequence of diagonal/near-diagonal Padé approximants and hence as one approaches the SNBP the convergence behavior of Padé approximants degrades. For example, for the IEEE 118-bus system using the matrix method, with 61 terms one can converge at load-levels up to 98.2% of the SNBP, while with 201 terms one converges up to 99.8% of the SNBP. However, with 301 terms one can converge only up to 99.6% of the SNBP. One infers from these results that higher precision along with more terms aids in extending the region of numerical convergence. (Theoretically, when using diagonal Padé approximant, the convergence domain coincides with the function's domain, assuming an infinite series and infinite precision is used.) The primary loss of accuracy in the HEPF occurs during the calculation of the Padé approximants [16]. This manuscript compares eight different techniques for obtaining rational approximants or enhancing the numerical convergence properties, along with additional techniques involving algebraic as well as integral Hermite-Padé approximants all numerically tested on power series obtained by HEPF. However at loading levels that are significantly short of the SNBP, numerical convergence is typically not an issue.

Given that the numerical Achilles heel of HEPF (and the holomorphic embedding method (HEM) in general) is the calculation of the Padé approximant, this paper first focuses on the numerical performance of the most researched Padé-approximant algorithms for providing analytic continuation. Next we focus on the most researched convergence-accelerating algorithms that have no supporting theory, like Stahl's theorem, governing theoretical convergence.

The rest of this paper is organized as follows: Section 2 has descriptions of eight different ways of accelerating the convergence of a given power series, some of which are equivalent to diagonal/near-diagonal Padé approximants and hence are theoretically (though not numerically) guaranteed by Stahl's theorem to converge to the solution, while others have no theoretical convergence guarantee. Section 3 contains the numerical results for these different algorithms when applied to the IEEE 14-bus, 118-bus, 300-bus systems and a 6057-bus ERCOT system. Section 4 has brief descriptions of the Hermite-Padé approximants and Section 5 contains a discussion on why the quadratic approximants can give the exact solutions for the two-bus power-flow problem. Section 6 contains the numerical results for different systems using quadratic approximants. Finally, the conclusions are presented in Section 7.

2. Different methods of obtaining rational approximations for a power series

2.1. Algorithms that yield analytic continuation via Padé approximants

2.1.1. Algorithms that yield diagonal/near-diagonal Padé approximants

From Stahl's convergence theory, the diagonal/near-diagonal Padé approximants yield the maximal analytic continuation (analytic continuation over the maximal domain of the function) and hence provide the convergence guarantee relied on by HELM. A diagonal Padé approximant for a series with a finite number of terms is a rational approximant whose numerator and denomina-