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General equations of lumped parameter ladder circuits and a special approach to analyzing electrical line transient states

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ABSTRACT

The presented analytical procedure gives the equations that can be named as the General Equations of Lumped Parameter Ladder Circuits. The procedure is based on representing an electrical line through its lumped parameter model, applying the principle of superposition, and summation of the finite and infinite geometric series. The developed analytical procedure results in relatively simple analytical expressions for the relationship between currents and voltages at different points of transmission line models in both transient and steady state conditions. Also, the developed analytical procedure follows one by one all the phases of the actual physical process occurring during transient states in transmission lines. As such, this procedure is convenient for determination of power frequency over-voltages during a transient state of a transmission or distribution line. As an illustration of this, the paper presents a numerical example of determination of switching over-voltage at the open end of a transmission line.

1. Introduction

The behavior of voltages and currents along a transmission line is described by linear partial differential equation, the celebrated Telegrapher's equations. The derivation of these equations should be only the introduction to a study of transmission lines; the important thing is their solution.

The existence of sinusoidal oscillations covering a large range of frequencies, and development of modulators allowing information to be placed on such a signal, have led to the complete development of the solution of Telegrapher's equations for sinusoidal time function, known as the General Line Equations [1]. However, they are convenient only for analyses of the transmission lines operating in steady state regimes. In other words, by applying those influences and mutual relationships between parameters relevant to resonant phenomena and transient effects on power lines, remain rather fogy, or completely veiled.

It is well-known that lumped parameter circuits represent universal physical and mathematical models of systems having distributed parameters. They are useful in performing the analyses of behaviors of electrical and electronic systems, as well as in analyzing the phenomena in heat transfer, acoustics, etc. Since in the current engineering practice some of the ladder circuits have elements discrete by nature, e.g. circuit formed by transmission line ground wires, the lumped parameter ladder circuits are very adequate in these cases.

Using the lumped parameter model reduces the state space of the system to a finite dimension, and the partial differential equations of the continuous (infinite dimensional) time and space system into ordinary differential equations containing a finite number of parameters. In this way the considered physical quantities: voltages and currents become functions of time only. However, the final, closed-form (analytical) solution is not sufficiently accurate in the case of a lumped parameter ladder network formed in reality by transmission line ground wires and the belonging tower footing electrodes, especially in the cases when the number of the considered line spans is small.

During a ground fault in a power system, depending upon fault location, at one of the line ends, or somewhere along the line, one or two lumped parameter ladder circuit(s) is(are) formed through the ground wire(s) and tower grounding electrodes of the HV (high voltage) overhead line. Because of that, at one moment in the development of contemporary electric-power systems, the exact mathematical model of these circuits has appeared as a practical need.

Extensive work has been undertaken, especially over the last two decades of the twentieth century, to model transmission lines for the purpose of ground fault current analysis. The advantages and drawbacks of a great number of the developed methods and procedures have been discussed in detail in [2]. Here we shall only point out that in their developments and improvements two

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tendencies can be clearly distinguished. On one hand, an effort is made to make these methods more convenient for applications [3–6] due to a great number of cases that have to be analyzed. On the other hand, the authors try to improve the procedure further by including new, less important factors, but which, under some especially complex and unfavorable conditions, can contribute to obtaining an economically and technically acceptable solution [7,8].

Since 1960s many authors have presented methods for solving the ladder network formed by a transmission line ground wire(s). The approach generally used by these authors was to represent the line ground wire by an equivalent distributed parameter ladder circuit and to solve it by using the well-known General Line Equations. However, the proposed methods give sufficiently accurate results only when a line, or its section(s), represented and solved in this manner, has a sufficiently large number of spans [2].

The second group of the methods has been developed by using special matrix techniques relying on possibilities of modern computers. However, theoretical and practical considerations have shown that the relative error propagation can be a problem when solving a large complex matrix. Thus, the research efforts for overcoming this problem have been directed towards finding exact mathematical model, i.e. solution for lumped parameter ladder circuits.

In the course of 1980s, by using Kirchhoff's Laws, the principle of superposition, and the summation of geometric series, the equations were derived which take into consideration discrete nature of the lumped parameter ladder circuits [3,7,8]. However, the procedure is not finalized, so that the solutions obtained were only partial, i.e. valid only for certain specific cases, from the standpoint of the ladder circuit terminal conditions.

By deriving the General Equations of the Line Represented by Discrete Parameters [9], the problem of determination of general mathematical model for uniform discrete parameter ladder circuits seemed as if has definitively been solved. Also, since the development of the analytical procedure which successively follows one by one all the phases of the actual physical process occurring before steady state is established, the possibilities for obtaining a better insight into the physical nature of the resonant phenomena providing easy explanation and investigation of these in the transmission line and transformer windings were quite open [10–12].

After that, the mentioned general equations were used as theoretical foundation in many research papers concerning mainly the problems of determination of critical ground fault position [13], and ground fault current distribution along the lines that cannot be treated as homogenous [15]. Since the obtained calculation results were logical and within the expected limits, it looked as though the problem was correctly solved.

Unfortunately, the used analytical procedure because of the obvious oversight of the number 2 was not finalized properly and as a consequence of that the finally derived equations are not completely correct and presented in the final mathematical form. This certainly represents a serious impediment from the standpoint of their application and obtaining the correct results. Besides, the procedure of deriving these equations, given in Appendix of [9], indicates that it is of less importance. However, it has given a new foundation for an investigation of resonant phenomena [10–12] and opened a possibility for a new investigation of transient states that is to be presented here.

By introducing the corrections and completion in the derivation of the procedures in [9,13] this paper shows that the present approach represents, at the same time, an alternative for obtaining the well-known General Line Equations. The significance of this new approach lies in the fact that the development procedure strictly follows all the phases of the actual physical processes which precede establishing a steady state in a line. As such it can be used not only for the analysis of the resonant phenomena, but also for the analysis and determination of transient overvoltages. Also, as such developed analytical procedure has an undoubtedly value for educational purposes.

Since the General Line Equations are already known, as a contribution of this paper one could also consider obtaining on the, so far unknown, General Equations of Lumped Parameter Ladder Circuits. Also, additional contribution of the paper is the used analytical procedure which enables obtaining a direct insight into the reasons causing the resonant phenomena and transient states of electrical lines by using relatively simple analytical expressions.

2. Lumped parameter circuit of an electrical line

It is well-known that every electrical line for ac energy transmission or for telecommunications purposes can be represented by an arbitrary number of sections with lumped parameters. In this way, these lines, which in reality have distributed parameters, are fictitiously divided into an arbitrarily chosen number of equal sections with discrete parameters. The lumped parameter model of a line with the earth as the return conductor and with a total of N fictitious sections of discretization, from now on simply called "line sections", is presented in Fig. 1.

The notation has the following meaning:

U_0 – voltage representing external electromotive forces,
U_N – voltage appearing at the output of the line,
$I_0, I_1, \ldots, I_N$ – currents of individual line sections,
Z_N – load impedance, and
N – adopted number of line sections.

Here, it is necessary to mention that the quantities which represent phasors are written implying $e^{j\omega t}$ time dependence throughout.

As shown, the lumped parameter model of a homogeneous electrical line is composed of a chain of identical π-networks each containing one impedance Z_S and two admittances $Y_S/2$ which are located at the ends of each discretization section. Impedance Z_S and admittance Y_S are in the general case determined by

$$Z_S = zL/N = (r + j\omega\, l)L/N, \tag{1}$$

$$Y_S = yL/N = (g + j\omega c)L/N, \tag{2}$$

where
z – line impedance per unit length,
y – line admittance per unit length.

Parameters: r, 1, g and c represent per unit length series resistance, series inductance, shunt conductance and shunt capacitance of the line, respectively, while ω and L are the angular frequency of the sinusoidal voltage U_0 and total line length, respectively.

The parameters r, 1, g and c are dependent on the geometry and insulation of the conductor, the electric, dielectric, and magnetic properties of the materials used in constructing the line. The geometrical factors are independent of the frequency, but the physical properties (resistivity, dielectric constant, and permeability) are in general frequency dependent. Thus, for simplicity, we will consider here the electrical quantities only at a fixed frequency.

For the purpose of starting our analysis, let us assume that the arbitrarily chosen line section length is so small that the admittance $Y_S/2$ at the line ends (denoted by dashed lines) can be neglected. At the same time, it means that I_0 represents the sending/input current, while I_N represents the receiving/output current of the presented lumped parameter ladder circuit (Fig. 1). The lumped parameter network obtained by using this approximation