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Phase shifting transformer model for direct approach power flow studies

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ABSTRACT

This proposal is intended to extend the field of application of an extremely efficient power flow algorithm used in radial and weakly meshed grids, the so-called Direct Approach (DA) method. In this work the method is broadened with the possibility of handling shunt admittances, transformers with taps, and phase shifting transformers. While the integration of the two former elements in the DA solver is quite straightforward, the use of phase shifting transformers is far from obvious due to their inherent non-symmetrical admittance matrix. Thus, a model for phase shifting transformers is proposed in this contribution, which allows the use of the DA method in grids that include such devices. A set of case studies is conducted in the contexts of a balanced industrial grid and a standard testbed to demonstrate the validity of the proposal.

1. Introduction

Power flow solvers are an essential tool in the operation and planning of power systems. They allow the assessment of voltage profiles, power flows and losses in the grid, and thus, they are crucial to detect unacceptable voltage deviations and identify overloaded components. Furthermore, power flow algorithms are used to conduct reliability studies and foresee the impact of future demand [1,2].

The most traditional power flow methods such as Newton-Raphson and Gauss-Seidel, used widely in transmission systems, do not offer the best performance and robustness when applied to the distribution level [3]. This is due to the especial nature of the distribution network, characterized by a radial or weakly meshed topology and a high R/X ratio. Several approaches have been proposed in order to deal with these particular features, such as the implicit Z-bus Gauss method [4] and backward-forward sweep methods [5,6]. In the latter group, a very efficient formulation called the direct approach (DA) was proposed in [7]. The DA method avoids the time-consuming tasks of LU factorization and forward and backward substitution of the Jacobian or admittance matrices, which are a commonplace in traditional formulations. The characteristics of DA method make it ideal for real-time applications in the smart grid context. In [8], the DA solver is used in the core of an optimal power flow (OPF) algorithm to provide references to a distribution FACTS in an industrial grid. High update rates are needed in this type of applications and the DA solver accommodates perfectly to this requirement.

The three-phase approach used in [7] takes series self-impedances and mutual couplings into consideration; however, shunt admittances are neglected. Even if that assumption can be enough to run a power flow analysis at the lowest voltage levels of the distribution grid, characterized by short-length lines and untapped transformers, ignoring shunt admittances strongly limits the application of the method to higher voltage levels. The extension of the method to accommodate medium-length lines and transformers with tap changers in a balanced environment is presented in this paper. Though no previous references to this use have been found, its application is fairly straightforward.

In a pure radial grid, a post-processing of the voltage phase angles after the application of the power flow solver is enough to account for the transformer phase shift. However, if a weakly meshed grid is to be considered, this method is no longer valid. Thus, a model of the phase shifting transformer, both to consider specific devices used to control the active power flow in the loop and to include the phase shift of common power transformers, is mandatory. Modeling of phase shifting transformers in power flow studies is a non-trivial problem, as they cannot be represented by a pi-equivalent component due to their inherent asymmetric admittance matrix [1]. A set of different phase shifting transformer models is available for application in various fields of study, to both steady state [9–13] and transient simulation [14]. In [15], a survey on phase shifting transformer models for steady state analysis is presented; however, none of them are expressed in a suitable form to be embedded in the DA solver. In this work, a new model is proposed to overcome this limitation.

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The DA method, as described in [7], is presented in Section 2 for the benefit of the reader. Section 3 presents a straightforward method to include shunt admittances in the DA solver. Thus, those components capable of being represented by pi-equivalent models, such as medium-length lines and transformers with tap changers, can be easily included in the problem. In Section 4, the new phase shifting transformer model is presented together with minor modifications to be performed in the DA algorithm. Three case studies are presented in Section 5 in order to illustrate the implementation procedure and demonstrate the validity of the proposal. Finally, Section 6 summarizes the most important results of this study.

2. Direct approach power flow

The method to be proposed in this contribution is based on the DA formulation of the power flow problem [7]. This is a technique, especially designed for radial networks, inspired by well-known backward-forward sweep methods such as Ladder Iterative Technique [6]. DA provides a very compact vectorized formulation with excellent computational and convergence characteristics.

In the application of DA to balanced grids, lines and transformers are modeled as series impedances, z_{ik}, as it is shown in Fig. 1. The equivalent bus current injection vector, $\boldsymbol{I_g}$, is calculated from the power injection at each bus, i, given the estimation of the bus voltage vector $\boldsymbol{V}$ at iteration (n) as

$$I_{gi}^{(n)} = \frac{P_i - jQ_i}{conj(V_i^{(n)})}. \tag{1}$$

Assuming a radial grid, the branch current vector can be calculated as

$$\boldsymbol{B}^{(n)} = \mathbf{BIBC} \cdot \boldsymbol{I_g}^{(n)}, \tag{2}$$

where **BIBC** is the so-called bus-injection to branch-current matrix. The entry $BIBC_{bi}$ equals 1 if the current injection of node i contributes to the branch current B_b, and equals 0 otherwise. Finally, a better approximation to the voltage profile can be obtained from

$$\Delta\boldsymbol{V}^{(n+1)} = \mathbf{BCBV} \cdot \boldsymbol{B}^{(n)}, \tag{3}$$

where **BCBV** is the branch-current to bus-voltage matrix. The entry $BCBV_{ib}$ equals the series impedance of branch b if that branch is in the path from node i to the slack bus, and equals 0 otherwise. $\Delta\boldsymbol{V}$ is a vector with the voltage of the slack bus referred to the different bus voltages. An improved approximation to the state variables is subsequently obtained by

$$\boldsymbol{V}^{(n+1)} = \boldsymbol{V_s} - \Delta\boldsymbol{V}^{(n+1)}, \tag{4}$$

where $\boldsymbol{V_s}$ is a column vector with the slack bus voltage at each entry.

Starting from a flat voltage profile, the solution of the distribution power flow is reached by solving (1)–(4) iteratively up to a specified convergence threshold.

In order to include the treatment of meshes in the network, Teng [7] proposes minor modifications to be conducted in the definition of **BIBC** and **BCBV** and in the solution technique. A brief summary of these changes can be described as:

- Specific branches are selected to break the meshed grid into a radial network. Then, new entries are included in the current injection vector to account for the currents at the selected branches, i.e. $[\boldsymbol{I_g}\boldsymbol{B}_{\text{new}}]^T$.
- The **BIBC** matrix is built as in the base case, by considering the currents of the branches used to break the network as additional current injections. However, entries with the value -1 appear now to account for the contribution of the receiving node of the branches used to break the network due to the inverted current reference. Notice that the double-sided contribution of the sending and receiving nodes of a branch used to break the network, B_c, to the current of those branches upstream from the first common parent node, B_b, is null, as they have the same value but opposite references.
  Additionally, new rows are added to the **BIBC** matrix with a single non-null entry in order to identify the currents of the branches used to break the network. Taking all this into account the modified **BIBC** matrix can be obtained as

$$\begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{B}_{\text{new}} \end{bmatrix}^{(n)} = \mathbf{BIBC} \cdot \begin{bmatrix} \boldsymbol{I_g} \\ \boldsymbol{B}_{\text{new}} \end{bmatrix}^{(n)}. \tag{5}$$

- The **BCBV** matrix is built as in the base case, but a new row is added for each loop in the grid to account for KVL. The impedances included in the entries of the new rows of the matrix are signed positive or negative according to the reference of the current at the different branches. Then, (3) is reformulated as

$$\begin{bmatrix} \Delta\boldsymbol{V} \\ 0 \end{bmatrix}^{(n+1)} = \mathbf{BCBV} \cdot \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{B}_{\text{new}} \end{bmatrix}^{(n)}. \tag{6}$$

- By using (5) and (6) and rewriting the resulting matrix, it follows that

$$\begin{bmatrix} \Delta\boldsymbol{V} \\ 0 \end{bmatrix}^{(n+1)} = \mathbf{BCBV} \cdot \mathbf{BIBC} \cdot \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{B}_{\text{new}} \end{bmatrix}^{(n)} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{P} \\ \boldsymbol{M} & \boldsymbol{N} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{B}_{\text{new}} \end{bmatrix}^{(n)}.$$

The application of Kron reduction to (7) leads to

$$\Delta\boldsymbol{V}^{(n+1)} = (\boldsymbol{A} - \boldsymbol{M}^T\boldsymbol{N}^{-1}\boldsymbol{M})\boldsymbol{I_g}^{(n)}. \tag{7}$$

The iterative use of (1), (7) and (4), in this order, allows the application of the DA method to weakly meshed grids.

3. Including pi-equivalent models

The DA method in [7] models the lines and transformers in balanced systems by simple series impedances. While this is acceptable for short-length lines and untapped transformers, minor

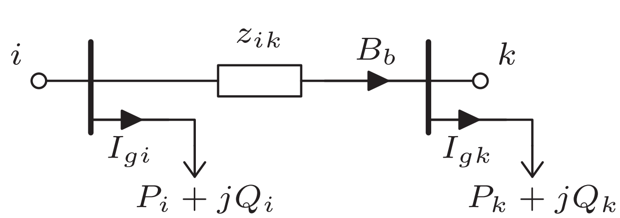

Fig. 1. Scheme used in the DA method.

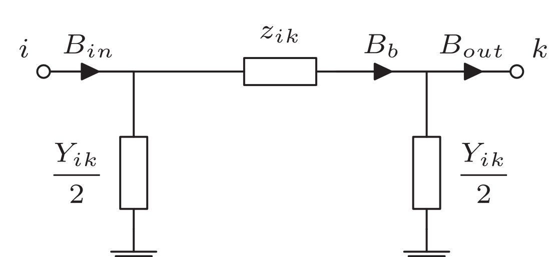

Fig. 2. Pi-equivalent line model.