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On the problem of optimal estimation of balanced and symmetric three-phase signals

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ABSTRACT

This paper revisits the fundamental problem of optimal estimation of the magnitude and phase of balanced and symmetric three-phase voltage or current signals. We analyze and compare various setups for the corresponding optimal Kalman filter, including the direct use of three-phase measurements, as well as measurements subjected to the Clarke transform in real or complex form. One contribution is to show that the standard practice of disregarding the transformed zero-component of the Clarke transformed three-phase signal almost always leads to a sub-optimal performance of the Kalman estimator. Our analysis extends to show that the closely related complex Kalman estimator is also sub-optimal and that optimal performance can be recovered if the zero-component is made available to the filter provided that the noises are properly characterized. These results are illustrated by means of simple numerical examples, which also highlight the importance of correctly modeling the noise characteristics if a real or complex form of the Clarke transformation is to be used. We conclude the paper with a unified set of guidelines or best practices regarding the use of optimal Kalman estimators for balanced and symmetric three-phase signals.

1. Introduction

Techniques for monitoring and control of power systems have been continuously evolving, and in the growing scenario of distributed generation and efficient microgrids, information extracted from measurements is crucial. Sophisticated applications, such as fault location and parameter identification [1–5], use information extraction techniques on measurements obtained from *Phasor Measurement Units* (PMUs) and digital relays [6,42]. In these applications, three-phase signals must be estimated from noisy measurements [7]. Therefore, estimating the parameters of three-phase sinusoidal signals, such as the magnitude, phase angle, and frequency of the fundamental component is imperative to the control and protection of power grids [8].

Noise has been a concern in power systems because its presence impacts the precision of the estimates and the estimation dynamics. In the IEEE C37.118.1 standard for synchrophasors measurements it is explicitly stated that the existence of undesirable noise components in the input signals of measurement equipment causes measurement errors and affects the tracking of power and dynamic phenomena [9]. Many techniques for estimating parameters of single-phase and three-phase sinusoidal signals were proposed in the literature. The work [6] suggests that most estimation algorithms are DFT-based but that in order achieve improved estimation performance many authors have proposed algorithms based on the Kalman filter [10–19], least square estimation [20,16], phase locked loop (PLL) [21–24], recursive wavelets [25], and intelligent systems [26]. It is well known that the estimation of amplitude and phase, more commonly denominated, phasor estimation, is the basis for estimating other important parameters of a signal, such as the fundamental frequency [18]. Estimates of the phasor and frequency are most of the time obtained jointly, even though some have suggested that separate estimates might be preferred to avoid instabilities when the system frequency moves far away from the fundamental frequency [27].

Numerous works [28,29,27,30,6] deal with comparing the performance and computational burden of different estimation algorithms and point out the advantages and drawbacks of each one. Some works apply these algorithms in real PMU data, such as in [31–34,17,35]. In this paper, we focus on phasor estimation methods based on the Kalman filter, with implications to possibly enhanced versions and variations. Kalman filtering provides a

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framework for the optimal estimate of the state of a linear system given an incomplete or complete set of noisy measurements [36,37,6].

In the case of estimation of balanced and symmetric three-phase signals, the state is a representation of the fundamental frequency phasor. Background on three-phase signals and optimal Kalman estimation is revisited in Section 2. Various setups for the application of the Kalman filter on the problem of estimation of balanced and symmetric three-phase signals are presented in Section 3.

Most works on phasor and frequency estimation apply the Kalman filter after transforming the noisy three-phase measurements into two orthogonal signals by using the classical Clarke transform. The Clarke transform applied to a purely sinusoidal balanced and symmetric three-phase signal produces a set of two orthogonal signals, the α and β components, and one identically zero signal, the zero-component [38]. For this reason, it is common practice to ignore the zero-component of measurements that have been Clarke transformed when performing optimal estimation. For example, the following papers make use of such approach when setting up the Kalman filter (with possible enhancements and variations) on the real [11,39,14] or complex [10,18,17,6,19] domains.

However, the Clarke transformation of noisy three-phase signals does not result in an identically null zero-component. Our first contribution is to show that discarding the noisy zero-component often leads to sub-optimal estimation performance. Remarkably, the Kalman filter is able to make use of the information present in the noisy zero-component, even if no purely sinusoidal component is present. This statement is made precise in Theorem 1 to be presented in Section 4. In Section 5, we show that this loss of optimality also extends to the case where complex Clarke transformed measurements are used. In Section 6, we show that optimal performance of the Kalman estimator when using Clarke transformed three-phase measurements can be recovered if the noisy zero-component is made available to the filter provided that the noises are properly characterized. This is the subject of Theorem 2.

Although our focus is on phasor estimation, the results and conclusions obtained in the present paper should continue to hold in more complex estimation tasks, such as, for example, that of joint frequency estimation, the estimate of higher harmonics, or the estimate of unbalanced signals using symmetric components. In the case of frequency estimation, several authors have reported success with estimating the frequency using nonlinear versions of the Kalman filter [17,28]. As the frequency is derived from the stationary phasors [9,6] and the nonlinear estimators determine the covariance by linearizing the model, the loss of information and consequent sub-optimal estimation that arises when one ignores the zero-component measurement will extend also to such frequency estimators. In the same vein, the present results will also be key for a more comprehensive theory that addresses the estimation of positive and negative sequences in unbalanced and asymmetric signals, which we will deal with in future works.

We close the paper with Section 7, in which we present a unified set of guidelines or best practices regarding the use of optimal Kalman phasor estimators for balanced and symmetric three-phase signals, followed by brief conclusions in Section 8.

2. Problem statement and background

Consider the set of purely sinusoidal three-phase continuous-time signals:

$$y_{abc}(t) = \begin{pmatrix} y_a(t) \\ y_b(t) \\ y_c(t) \end{pmatrix} = \begin{pmatrix} Y_a\cos(\omega t+\phi_a) \\ Y_b\cos(\omega t+\phi_b) \\ Y_c\cos(\omega t+\phi_b) \end{pmatrix}, \tag{1}$$

representing phase or line voltages and/or node currents in a three-phase electrical circuit and the three-phase noisy measurement of such signals:

$$y_{ABC}(t) = \begin{pmatrix} y_A(t) \\ y_B(t) \\ y_C(t) \end{pmatrix}, \tag{2}$$

where the subscripts a, b, c and A, B, and C, represent the nodes where the measurements take place.

We assume two distinct sources of noise in the noisy measurements: the first is the *common-mode* scalar noise signal $r(t)$, which affects all components equally; the second is the *per-component* noise vector

$$z_{abc}(t) = \begin{pmatrix} z_a(t) \\ z_b(t) \\ z_c(t) \end{pmatrix},$$

which can be different at each measurement node. The combined signal model is given by the vector equation

$$y_{ABC}(t) = y_{abc}(t) + r(t) + z_{abc}(t). \tag{3}$$

This generic setup can be used, for instance, to represent the three-phase signals in Fig. 1.

In this paper, we shall revisit optimal estimation procedures with the intent of recovering the noise-free signals (1). More specifically, amplitudes Y_a, Y_b, Y_c, and phases ϕ_a, ϕ_b, and ϕ_c, from the set of corrupted measurements (2) under the balanced and symmetrical scenario, i.e., on the presence of the following assumptions:

Assumption 1 (*Symmetric three-phase signals*). Phases in the signals of vector $y_{abc}(t)$ are shifted by exactly $2\pi/3$ from each other, that is:

$$\phi_a = \phi_1, \quad \phi_b = \phi_1 + \frac{2\pi}{3}, \quad \phi_c = \phi_1 - \frac{2\pi}{3}.$$

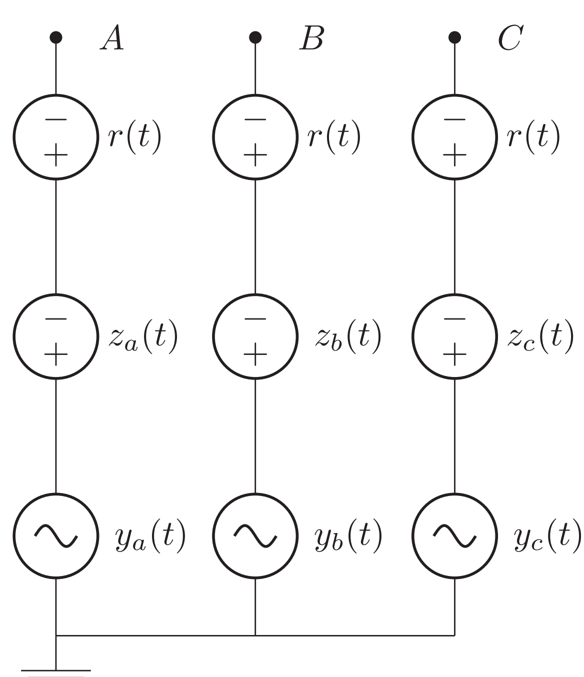

Fig. 1. Three-phase signals subjected to noise.