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A stochastic methodology to adjust controllers based on moments Lyapunov exponents: Application to power systems

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ABSTRACT

This paper presents a methodology, based on linear analysis, that uses stability indices based on the calculation of moments Lyapunov exponents to determine the optimum adjustment of controllers in linear stochastic systems. The purpose is to define what values the parameters must have so that the system, subjected to perturbations self-sustained over time, can maintain its stability during operation in permanent regime. The proposed development is applied to three test systems, including two electric power system.

1. Introduction

One of the largest problems associated with the operation of systems in engineering consists in determining the operating conditions required to maintain the stability when perturbations or situations that may cause collapse occur. Traditionally, the use of controllers ensures the system's stability under different operating conditions, and for that reason the optimum adjustment of the parameters is a very important matter to be solved. The techniques used to determine the optimum values of these devices are generally based on tools that do not consider the system's evolution over time, i.e., they use the deterministic approach.

Operating points defined arbitrarily by the operator are considered for the adjustment, but they do not necessarily represent the system's real dynamics. Furthermore, the traditional approach is based on the fact that the system's variation over time is not represented, and sometimes this does exactly fit reality.

The deterministic approach has certainly had excellent results in different applications, but it is necessary to incorporate methodologies that focus on the need to represent various operating conditions that have self-sustained dynamics over time. An example of this appears in the operation of Electric Power Systems: the incorporation of non conventional renewable energy sources, variation of consumption, changes in the parameters of the lines at the transmission level, to mention some. As a function of these data, there is the need to model this behavior at the time of adjusting the parameters of the controllers used in different applications, which allow the stability of the systems during the operation to be maintained. The work done presents diverse applications [1–3] where Lypunov exponents have been used to define the stability criteria in systems with a multiplicative perturbation model, from which adjustment proposals have been made considering dynamics sustained over time. In the particular case of [4], stationary distributions have been used to define stability criteria, when the perturbation model is additive, to then tune the parameters of a controller that have been determined by traditional techniques [5–7].

In [8] the calculation of the moments associated with Lyapunov exponents is presented as a stability criterion. It is shown and verified that moment $p = 0$ coincides with the almost sure Lyapunov exponent. However, no strategy for adjusting controller parameters is shown. In the present paper a methodology, based on linear analysis, is described that allows determining safety regions associated with the controllers, defining optimum gains for every fixed perturbation size. The methodology is applied to the three-dimensional oscillator, the infinite machine-busbar and the three machines electric power system.

The paper is structured as follows: Section 2 gives the mathematical foundations and models used to represent an electric power system as a stochastic linear system. Section 3 presents the results obtained for cases presented. Finally, Section 4 includes the conclusions and future work.

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2. Mathematical foundations

2.1. Model of the system

A dynamic system can be described by a nonlinear differential equation of the form $\dot{y}(t) = f(y(t), \xi(t,\omega), b)$ in $\mathbb{R}^d$ that is subjected to a perturbation sustained over time $\xi(t,\omega)$. Let us assume that b represents the parameter of a controller, which must be adjusted to decrease the effect of the random perturbations self-sustained in time.

To do the above, let us assume that an equivalent linear representation around an equilibrium point y^* of the system can be obtained, which has the following form:

$$\dot{x}(t) = A(\xi(t,\omega), b)x(t) \quad \text{in } \mathbb{R}^d \tag{1}$$

where $A(\xi(t,\omega))$ is the Jacobian matrix of the nonlinear system $f(y(t), \xi(t,\omega))$ at the equilibrium point y^*, and $b \in \mathbb{R}^d$ is a set of parameters that represent the controller. We will denote by $\varphi(t,x,\omega)$ the trajectories of (1), obtained from an initial condition $\varphi(0,x,\omega) = x \in \mathbb{R}^d$.

In relation to the perturbation, traditionally white noise or variations of this process have been considered as representations of $\xi(t,\omega)$, see [9]. In our case, let η be a process defined by the differential Eq. (2)

$$d\eta = X_0(\eta)dt + \sum_{i=1}^{r} X_i(\eta) \circ dW_i \quad \text{on } M \tag{2}$$

where $X_0, \ldots, X_r$ correspond to the vector fields C^∞ and "$\circ$" denotes the Stratonovic type stochastic differential equation, which more appropriate than Ito type to perform calculations.

Thus, the system perturbation ξ_t in (1) is modeled as a function of the background noise η_t in the form $\xi_t = f(\eta_t)$, with f a C^∞ function. For more details in relation to the noise model, see Ref. [8].

2.2. Lyapunov exponents and moments

The stability of system (1), by means of Lyapunov exponents has been developed in [3]. Each Lyapunov exponent for a given trajectory $\varphi(t,x,\omega)$, is obtained from

$$\lambda(x,\omega) = \limsup_{t\to\infty} \frac{1}{t} \log|\varphi(t,x,\omega)|, \tag{3}$$

On the other hand, for $p \in \mathbb{R}$ the Lyapunov exponent for the pth-moment is determined as

$$g(p,x) = \limsup_{t\to\infty} \frac{1}{t} \log \mathbb{E}|\varphi(t,x,\omega)|^p. \tag{4}$$

For further details of the calculation of moments, se Ref. [8].

Let us now assume that we consider the dependence of a controller's parameters represented by $b \in \mathbb{R}^d$. In that case the Lyapunov exponent and the moments are determined from

$$\lambda(x,\omega,b) = \limsup_{t\to\infty} \frac{1}{t} \log|\varphi(t,x,\omega,b)|, \tag{5}$$

$$g(p,x,b) = \limsup_{t\to\infty} \frac{1}{t} \log \mathbb{E}|\varphi(t,x,\omega,b)|^p. \tag{6}$$

In relation to Eq. (5), Refs. [1,2] report results on how the Lyapunov exponents can be used to adjust the controllers' parameters in Electric Power Systems.

2.3. Stability indices based on the calculation of Moment Lyapunov Exponents

With the purpose of presenting the stability indices, let us rewrite the stochastic linear system of Eq. (1) in the following way:

$$\dot{x}^\rho(t) = A(\xi^\rho(t,\omega), b)x^\rho(t)x \in \mathbb{R}^d, \xi^\rho(t,\omega) \in U^\rho \subset \mathbb{R}^m, \rho \geqslant 0 \tag{7}$$

where ρ is a variable that amplifies or reduces the size of the perturbation and $b \in \mathbb{R}^d$ is a set of parameters that represent the controller. According to [3], the almost sure Stability Radius is defined as

$$r = \inf\{\rho \geqslant 0, \lambda(\rho) > 0\} \tag{8}$$

Keep in mind that $\lambda(\rho)$ represents the almost sure Lyapunov exponent of system (7). It corresponds to the smallest perturbation size that allows the linear system to remain stable.

On the other hand, considering (6) the pth-moment of the Stability Radius is defined as (see [8]):

$$r(p) = \inf\{\rho \geqslant 0, g^\rho(p) > 0\} \quad \text{for each } p \in \mathbb{R} \tag{9}$$

Eq. (9) indicates, for every moment, the smallest size of the perturbation that allows the system to remain stable. In [8] it is shown that (8) and (9) have the same value for $p = 0$.

From Eq. (9) and the existence of parameter $b \in \mathbb{R}^d$, we define the following set of Stability Radii:

$$r(b,\rho) = \{\rho \geqslant 0, b \in \mathbb{R}^d \mid \max\{p \geqslant 0, g^\rho(p,b) < 0\}\} \tag{10}$$

Eq. (10) allows getting a surface of Stability Radii. For all the possible combinations of (b,ρ) in (10), the system will be stable. From the standpoint of the system's control, according to the perturbation ρ that is specified, it will be possible to determine the maximum value that the parameter of the considered controller will have.

To get the set of elements indicated in (10), the $g^\rho(p,b)$ terms are calculated from (11), as indicated in [8].

$$\begin{aligned} g(p,x,b) &= \lim_{T\to\infty} \frac{1}{T} \log \mathbb{E}|\varphi(T,x,\omega,b)|^p \\ &\approx \frac{1}{\alpha}\frac{1}{T}\sum_{j=1}^{\alpha}\sum_{t=1}^{T} \log\left(\frac{1}{\beta}\sum_{i=1}^{\beta}\left|x_t^j(i,b)\right|^p\right). \end{aligned} \tag{11}$$

where α represents the initial conditions, β is the number of realizations of the perturbation $\xi(t,\omega), b$ are the controller's parameters, and ρ is the size of the perturbation.

If we consider that $b \in \mathbb{R}^d$, another indicator that will be useful to analyze the system's response consists of

$$r_{max}(\rho) = \{\rho \geqslant 0, \max\{g(b,\rho), \forall b \in \mathbb{R}^d\}\} \tag{12}$$

From now on, Eq. (12) will be called the Absolute Stability Radius. This indicator will make it possible to analyze the effect of the perturbation on the Stability Radii.

In Ref. [3] are presented three numerical methods to compute Lyapunov exponents. In this paper we have used Method I described in [3].

2.4. Perturbation model used for power systems

An Electric Power System can be described by the following differential algebraic equation system:

$$\begin{aligned} \dot{x} &= f(x,y) \\ 0 &= g(x,y) \end{aligned} \tag{13}$$