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Comparison of time-varying phasor and *dq*0 dynamic models for large transmission networks

Juri Belikov [a,b], Yoash Levron [c,*]

[a] Faculty of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa 3200003, Israel
[b] Department of Computer Systems, Tallinn University of Technology, Ehitajate tee 5, 19086 Tallinn, Estonia
[c] The Andrew and Erna Viterbi Faculty of Electrical Engineering, Technion—Israel Institute of Technology, Haifa 3200003, Israel

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ABSTRACT

In recent years, with increasing penetration of small distributed generators and fast power electronics based devices, the assumption of quasi-static phasors is becoming increasingly inaccurate. In order to describe fast dynamic behavior and rapid amplitude and phase variations, more accurate dynamic models based on the *dq*0 transformation are used. To better understand the differences between these two models, in this work we compare their relative accuracy when applied to large-scale transmission networks. In this light, the present work describes the two types of models using similar terminology, which is based on *dq*0 quantities. Based on this result, we show that quasi-static models may be obtained from *dq*0 models at low frequencies, and that there exists a frequency range in which quasi-static model approximates the *dq*0 model well. The obtained results allow to estimate the frequency after which the quasi-static model cannot accurately describe the system dynamics, and *dq*0 models should be used instead.

1. Introduction

Quasi-static models are a well-known approach for modeling the dynamics of large power systems. A key assumption in such models is that voltages and currents may be represented by time-varying phasors, since magnitudes and phases throughout the network change slowly in comparison to the system frequency [1,2]. The transmission network is represented in this case by a constant admittance matrix, and the system dynamics are described either in terms of phasor components, or using the power-flow equations. A key advantage of quasi-static models is relative simplicity, since large transmission networks can be described by purely algebraic equations [3]. In addition, since quasi-static models employ phasors instead of sinusoidal AC signals, the system operating point (or equilibrium point) is well-defined, a property which enables small-signal and stability studies. Due to these properties, quasi-static models have been extensively used in the analysis of dynamic interactions that occur in time frames of seconds to minutes, and have historically enabled studies of machine stability, inter-area oscillation, and other slow dynamic interactions that occur in large power systems [1,4–6].

In recent years, with increasing penetration of small distributed generators and fast power electronics based devices, the assumption of quasi-static phasors is becoming increasingly inaccurate [7]. In order to describe fast dynamic behavior and rapid amplitude and phase variations, more accurate dynamic models are being used, among them models that are based on the *dq*0 transformation [8]. The *dq*0 transformation maps three-phase signals in the *abc* reference frame to new signals in a *dq*0 reference frame, that often rotates with the electrical angle of a machine's rotor. This approach provides several important advantages when the system is balanced or symmetrically configured [9], one of them is that sinusoidal three-phase signals are mapped to constant signals at steady-state [10,11]. Similarly to quasi-static models, *dq*0-based models are often time-invariant, and allow to describe the system by means of ordinary differential equations, using standard state-space formalism. Since *dq*0 signals are constant at steady-state, the operating point of the system is well-defined, and as a result, the dynamic equations can be linearized, and a small-signal analysis can be performed [4]. For these reasons, the *dq*0 transformation is often used to evaluate the dynamics of symmetric or symmetrically configured units, and provides efficient tools for designing suitable controllers [12]. A review of simulation techniques based on *dq*0 quantities can be found in [10,13,14].

The *dq*0 transformation is increasingly used today when modeling distributed sources, complex loads, renewable generators, and

* Corresponding author.
E-mail addresses: juri.belikov@ttu.ee (J. Belikov), yoashl@ee.technion.ac.il (Y. Levron).

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power electronics based devices [15–18]. In addition, widespread use of this transformation for modeling large-scale power systems is still a pending issue, that is recently being explored. Toward this end, one central challenge is to combine various $dq0$ models that use local reference frames to form a complete dynamic model describing a large-scale power system. Another associated challenge is to provide a $dq0$-based model of general transmission networks, that will be of low complexity and easy-to-use. Several recent works exploring these questions are [19], which shows $dq0$ models of elementary passive components, work [9], that presents $dq0$-based models of three-phase networks with RL elements, work [20], that shows a nonminimal state-space model of networks with standard branches, and [21], that presents a frequency domain model of transmission networks using $dq0$ quantities.

Models based on $dq0$ quantities (as reviewed above) are known to extend the classic quasi-static model, and are generally considered to be more accurate. However, while quasi-static models are well-known and widely used, $dq0$ models are only now being explored in the context of large-scale power systems. To better understand the differences between these two models, in this work we compare their relative accuracy when applied to large-scale transmission networks. The two types of models are described using similar terminology, which is based on $dq0$ quantities. Based on this result, we show that quasi-static models may be obtained from $dq0$ models at low frequencies, and that there exists a frequency range in which quasi-static model approximates the $dq0$ model well. Results are demonstrated on the basis of several test-cases.

The paper is organized as follows. Section 2 presents the basic definition of $dq0$ quantities. Section 3 reviews quasi-static models and links them to $dq0$ quantities. The $dq0$ models of large-scale networks are presented in Section 4. Section 5 provides a comparison between the two models, followed by simulation results in Section 6. Concluding remarks are presented in Section 7.

2. Preliminaries

This section recalls the basic definition of the $dq0$ transformation. Consider a reference frame rotating with an angle of $\theta(t)$. For instance, in a synchronous machine, $\theta(t)$ is typically selected to be the rotor electrical angle. Let $\tilde{\zeta}$ represent the quantity to be transformed (current, voltage, or flux), and use the compact notation $\zeta_{abc} = [\zeta_a, \zeta_b, \zeta_c]^\top$, $\zeta_{dq0} = [\zeta_d, \zeta_q, \zeta_0]^\top$. The $dq0$ transformation with respect to the reference frame rotating with the angle θ can be defined as [8, Appendix C]

$$\tilde{\zeta}_{dq0} = T_\theta \zeta_{abc} \tag{1}$$

with

$$T_\theta = \frac{2}{3}\begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}. \tag{2}$$

3. Quasi-static models

The traditional quasi-static model is based on the assumption that the frequency of voltage and current signals throughout the network is approximately constant. As a result, AC signals can be modeled accurately enough by means of time-varying phasors, and the transmission network can be represented by a constant admittance matrix Y^{bus}. The transmission network is then modeled using the linear relation $I = Y^{bus}V$, which is equivalent to the well-known power flow equations [3].

To present the quasi-static models and $dq0$ models using similar terminology, we begin by developing the relationship between $dq0$ signals and time-varying phasors. Assume a unit with balanced three-phase AC voltages given by

$$\begin{aligned} v_a(t) &= A(t)\cos(\omega_s t + \psi(t)), \\ v_b(t) &= A(t)\cos\left(\omega_s t + \psi(t) - \tfrac{2\pi}{3}\right), \\ v_c(t) &= A(t)\cos\left(\omega_s t + \psi(t) + \tfrac{2\pi}{3}\right). \end{aligned} \tag{3}$$

Assuming that variations in the magnitude $A(t)$ and phase $\psi(t)$ are slow in comparison to the frequency ω_s, these voltages may be represented by the time-varying phasor $V(t) = \frac{A(t)}{\sqrt{2}} e^{j\psi(t)}$. In addition, based on the $dq0$ transformation (19), equivalent $dq0$ voltages with respect to the reference frame $\omega_s t$ are

$$\begin{aligned} v_d(t) &= A(t)\cos(\psi(t)), \\ v_q(t) &= A(t)\sin(\psi(t)), \\ v_0(t) &= 0. \end{aligned} \tag{4}$$

It immediately follows that a time-varying phasor relates to $dq0$ quantities as

$$V(t) = \frac{A(t)}{\sqrt{2}} e^{j\psi(t)} = \frac{A(t)}{\sqrt{2}}(\cos(\psi(t)) + j\sin(\psi(t))) = \frac{1}{\sqrt{2}}\left(v_d(t) + jv_q(t)\right), \tag{5}$$

or alternatively,

$$\begin{aligned} v_d(t) &= \sqrt{2}Re\{V(t)\}, \\ v_q(t) &= \sqrt{2}Im\{V(t)\}, \end{aligned} \tag{6}$$

and the same relations hold for currents. Let us now examine the active and reactive powers. Assuming a voltage phasor $V(t)$ and a current phasor $I(t)$, the powers are given by

$$\begin{aligned} P(t) &= Re\{V(t)I^*(t)\} = \tfrac{1}{2}\left(v_d(t)i_d(t) + v_q(t)i_q(t)\right), \\ Q(t) &= Im\{V(t)I^*(t)\} = \tfrac{1}{2}\left(v_q(t)i_d(t) - v_d(t)i_q(t)\right). \end{aligned} \tag{7}$$

The first equation in (7) shows the dual meaning of the active power $P(t)$. In the specific case of a static or quasi-static system, in which transients are slow in comparison to ω_s, the active power $P(t)$ of each phase is the average power over a line cycle. In addition, the instantaneous sum of powers for the three-phases is $P_{3\phi}(t) = 3P(t)$. Note that if the system is not quasi-static (for instance, during a fast transient) the average powers P and Q are not well-defined. However, the expression for $P_{3\phi}(t)$ still holds.

Using time-varying phasors, the network is usually described by the nodal admittance matrix Y^{bus}. Assume a network with N buses, and denote the bus voltages by $V(t) = [v_1(t), \ldots, v_N(t)]^\top$, and the bus injected currents by $I(t) = [i_1(t), \ldots, i_N(t)]^\top$. All quantities in these vectors are time-varying phasors. The network is then described by the relation

$$I(t) = Y^{bus}V(t). \tag{8}$$

This is a *quasi-static* model of the transmission network, since it is based on the assumption that phasors change slowly in comparison to ω_s. Due to this assumption, the frequency throughout the network is approximately constant, and as a result, the matrix Y^{bus} is composed of constant admittances, which are computed at the fixed frequency ω_s, such that $Y^{bus} = Y^{bus}(j\omega_s)$. Eq. (8) can also be written in terms of its real and imaginary parts as

$$\begin{aligned} Re\{I(t)\} &= Re\{Y^{bus}\}Re\{V(t)\} - Im\{Y^{bus}\}Im\{V(t)\}, \\ Im\{I(t)\} &= Im\{Y^{bus}\}Re\{V(t)\} + Re\{Y^{bus}\}Im\{V(t)\}, \end{aligned} \tag{9}$$

or alternatively, using the power flow equations

$$\begin{aligned} P_n(t) &= |V_n(t)|\sum_{k=1}^{N}|y_{nk}||V_k(t)|\cos(\angle y_{nk} + \delta_k - \delta_n), \\ Q_n(t) &= -|V_n(t)|\sum_{k=1}^{N}|y_{nk}||V_k(t)|\sin(\angle y_{nk} + \delta_k - \delta_n), \end{aligned} \tag{10}$$