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Animations of electromagnetic transients in power transmission lines by means of the two-dimensional numerical Laplace transform

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ABSTRACT

This paper presents the use of animations of the wave propagation phenomena on power transmission lines as a valuable tool for research and educational purposes. These animations are generated from data obtained by transient analysis in the frequency domain applying the two dimensional numerical Laplace transform. Since the line electrical parameters are defined in the frequency domain, the frequency dependence of such parameters is directly taken into account. These animations allow an insightful illustration of how travelling waves propagate along transmission lines during an electromagnetic transient, making them a useful tool to determine transient overvoltages that occur at internal points along power transmission lines. In the educational field, using these animations in electrical engineering courses can help students to have a better understanding of the traveling wave concept and how it differs from the circuit theory. In addition, it allows detailed comparison of the wave propagation along lines under different conditions, such as type of excitation, type of load, introduction of losses, etc.

1. Introduction

The wave propagation along transmission lines is defined by means of a set of partial differential equations in the space-time domain, known as the Telegrapher equations. Previous work showed that the application of the Laplace transform in both the space and time dimensions results in a set of algebraic equations in the *q-s* domain (spatial frequency-temporal frequency) which are substantially easier to solve that the original set of PDEs [1,2]. The subsequent use of the inverse two dimensional numerical Laplace transform algorithm allows the computation of transient voltage and current profiles along the lines. These profiles provide detailed information about the propagation of the voltage and current waves under different transient conditions. An example of a voltage transient profile is presented in Fig. 1. This figure shows the transient voltage profile of a 1000 m transmission line with the receiving end open and an ideal unit step voltage source connected at the sending end.

This paper presents the application of the technique described above to generate animations of electromagnetics transients on transmission lines. The simulation data obtained by using the 2DNLT is stored in three-dimensional matrices that contain all the necessary information to generate the animations presented in this work. These animations allow a very detailed and interactive study of travelling waves propagating along a transmission line during an electromagnetic transient, as well as localization of overvoltages appearing at interior points of the line. This type of study is not possible using traditional transient simulation software, since this kind of software usually provides transient responses only at specific nodes or branches.

The animations are generated using MATLAB, which allows easy manipulation of the three dimensional matrices that contain the data of the transient profiles, produces high quality animations that can be saved in GIF file format, does not require advanced programming knowledge, and is widely used around the world allowing this tool to be easily applied or even reproduced.

Previous authors have introduced tools for the animation of electromagnetic transients [4–7]. However, none of the previous works considers the transmission line model in the frequency domain or the animation of transients due to incident electromagnetic fields (illuminated line) [3]. Therefore, the main contributions of this work are as follows:

Abbreviations: 2DNLT, two-dimensional numerical Laplace transform; PDE, partial differential equation.

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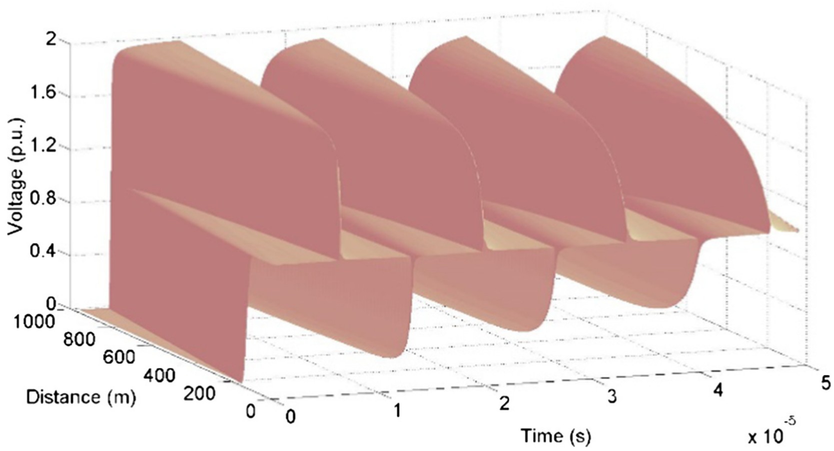

Fig. 1. Transient voltage profile along a transmission line.

1. The extension of the 2DNLT, presented in [1,2] to produce animations of transients in transmission systems.
2. The application of a frequency domain method, which allows producing animations of the wave propagation along full-frequency dependent transmission lines and comparing the results with ideal conditions.
3. The possibility to generate animations of transmission lines excited by an external electromagnetic field, such as an indirect lightning stroke.

2. Computation of voltage and current profiles along a transmission line

This section briefly describes the application of the Laplace Transform to obtain and solve the transmission line model in the *q-s* domain. A more detailed explanation can be found in [1,3].

2.1. Solution of the telegrapher equations in the q-s domain

The telegrapher equations for a uniform multiconductor line are defined in the *z-s* domain (space-temporal frequency) in matrix form as follows:

$$\frac{d}{dz}\begin{bmatrix}\mathbf{V}(z,s)\\ \mathbf{I}(z,s)\end{bmatrix}=\begin{bmatrix}\mathbf{0} & -\mathbf{Z}\\ -\mathbf{Y} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}(z,s)\\ \mathbf{I}(z,s)\end{bmatrix} \tag{1}$$

where $\mathbf{V}(z,s)$ and $\mathbf{I}(z,s)$ are voltages and currents along the propagation axis z, $\mathbf{Z}$ and $\mathbf{Y}$ are the series impedance and shunt admittance of the line, respectively. By applying the Laplace transform to (1) with respect to the *z* variable and solving for the voltage and currents along the line, the following equation is obtained:

$$\begin{bmatrix}\mathbf{V}(q,s)\\ \mathbf{I}(q,s)\end{bmatrix}=\begin{bmatrix}q\mathbf{U} & -\mathbf{Z}\\ -\mathbf{Y} & q\mathbf{U}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{V_0}(s)\\ \mathbf{I_0}(s)\end{bmatrix} \tag{2}$$

where $[\mathbf{V}(q,s) \quad \mathbf{I}(q,s)]^{\mathrm{T}}$ are the voltages and current profiles along the line in the *q-s* domain (spatial frequency-temporal frequency), $\mathbf{V_0}(s)$ and $\mathbf{I_0}(s)$ are the voltage and current at the beginning of the line in the *s* domain, respectively, and $\mathbf{U}$ is the identity matrix. From (2), it can be noted that the telegrapher equations have been reduced to an algebraic form in the *q-s* domain. In order to solve (2) it is necessary to calculate the line parameters and the voltage and current at the beginning of the line in the *s* domain. In this paper, the parameter calculation is done through the method of images and the frequency dependence of the line's parameters is taken into consideration by means of the concept of complex penetration depth [8]. The voltage and current at the beginning of the line can be computed as shown in [1].

2.2. Inclusion of an external electromagnetic field

As mentioned in Section 1, the procedure to generate animations can accommodate the case of a line excited by an incident electromagnetic field, such as the one produced by an indirect lightning stroke. According to Taylor's formulation [9], an external electromagnetic field exciting a transmission line can considered by means of voltage and current sources distributed along the line. The telegrapher equations in the *z-s* domain (Eq. (1)) can be rewritten as follows to include the effect of such fields:

$$\frac{d}{dz}\begin{bmatrix}\mathbf{V}(z,s)\\ \mathbf{I}(z,s)\end{bmatrix}=\begin{bmatrix}\mathbf{0} & -\mathbf{Z}\\ -\mathbf{Y} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}(z,s)\\ \mathbf{I}(z,s)\end{bmatrix}+\begin{bmatrix}\mathbf{V_F}(z,s)\\ \mathbf{I_F}(z,s)\end{bmatrix} \tag{3}$$

where $\mathbf{V_F}(z,s)$ and $\mathbf{I_F}(z,s)$ correspond to the distributed series voltage and shunt current sources along the line. For an indirect lighting stroke, these sources are computed using the formulae derived by Master and Uman [10]. A more detailed description of the computation of the sources in the *z-s* domain can be found in [11]. Applying the Laplace transform to (3) with respect to the *z* variable and solving for the voltage and currents along the line, the following equation is obtained [3]:

$$\begin{bmatrix}\mathbf{V}(q,s)\\ \mathbf{I}(q,s)\end{bmatrix}=\begin{bmatrix}q\mathbf{U} & -\mathbf{Z}\\ -\mathbf{Y} & q\mathbf{U}\end{bmatrix}^{-1}\left(\begin{bmatrix}\mathbf{V_0}(s)\\ \mathbf{I_0}(s)\end{bmatrix}+\begin{bmatrix}\mathbf{V_F}(q,s)\\ \mathbf{I_F}(q,s)\end{bmatrix}\right) \tag{4}$$

As in (2), to solve (4) it is necessary to calculate the voltage and current at the beginning of the line, as described in [3].

2.3. Transformation to the z-t domain

Vectors $\mathbf{V}(q,s)$ and $\mathbf{I}(q,s)$ from (2) and (4) are transformed to $\mathbf{v}(z,t)$ and $\mathbf{i}(z,t)$, which define transient voltage and current profiles along the line. This transformation is done numerically with the inverse 2DNLT algorithm. A detailed description of the implementation of the numerical Laplace transform algorithm for the analysis of transients in transmission lines can be found in [12,13] and its implementation for the transformation from the *q-s* domain to the *z-t* domain can be found in [1,14]. The use of the transmission line model defined in the *q-s* domain in conjunction with the 2DNLT algorithm has demonstrated very accurate results when compared at discrete points along the line with results from traditional transient simulation software, as shown in [1,14].