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A novel correlated intervals-based algorithm for distribution power flow calculation

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ABSTRACT

This paper proposes a novel correlated intervals-based algorithm for calculation of distribution power flows. The non-conservative interval operations are proposed. It takes into account the uncertainties of renewable resource-based generations (wind generators, solar panels and other) and distribution network loads. Their correlations are calculated from historically recorded patterns of input variables and directly integrated into the adopted forward-backward distribution power flow algorithm. The proposed algorithm is tested on two distribution network test examples: (1) 6-bus with the aim of detailed illustration of some algorithmic steps, and (2) 1003-bus for verification of global performances of the proposed algorithm. Obtained results are verified by Monte Carlo sampling and power flow calculations.

1. Introduction

Power flow calculation is a necessary tool for analysis, operation and planning of distribution networks. There are several types of algorithms for symmetrical power flow calculations in distribution networks: currents summation [1], powers summation [2], admittances/currents summation [3], algorithm based on network modeling by using the loop relations [4] and many others. On the other hand, the asymmetrical distribution three-phase power-flow models are presented in [5,6], while the DC-based optimal power flow (OPF) algorithm is proposed in [7]. All the mentioned references are classical power flow calculation algorithms; in other words, they are deterministic ones. However, there has been an increasing number of renewable and distributed resources included in distribution networks recently, such as wind generator-based farms, solar panels, and micro/mini/small hydro power plants [8]. The generation from these resources is stochastic in nature and they introduce a significant uncertainty in total production of distribution network. Similarly, input loads in load flow algorithms are often or usually provided from load forecast and thus, due to a possible forecasting error, these loads are also uncertain. Therefore, the power flow calculation of distribution networks need to consider the uncertainty being the consequence of included renewable resources and uncertain loads. It is difficult to predict exact active/reactive injections from renewable resources and loads. Ref. [9] considers the uncertainty of load, energy price, and power generated by renewable distributed generators (DGs). A fuzzy distribution power flow algorithm [10,11] can handle a few uncertainties in input variables (generations and loads). Ref. [12] also uses the fuzzy number arithmetic for DC power flow problem. Ref. [13] presents an OPF method that makes the power system robust to the uncertainty in operating conditions, caused by loads and renewable production. Ref. [14] considers the set-theoretic approach to the uncertainty modeling, where the uncertainty of renewable resources can be viewed as a forecasting error. It is bounded around the nominal forecast with some tolerance bounds.

Modern distribution networks for their optimal operation typically use historical and weather data. Also, operators have knowledge about load characteristics similarity and increased volume of real-time measurements in MV part and smart meters in LV part of distribution network. Some of these inputs are used to obtain the pseudo measurements for improving the network observability. Depending on the measurements accuracy and placement, the real-time and pseudo measurements uncertainty can be mutually correlated. These problems have been addressed in Refs. [15–18].

The appropriate models taking into account the parameter- and variable-based uncertainty are interval arithmetic based ones [19–23]. This is an elegant tool for a practical work with error bounds and uncertainties, and more generally with certain convex and bounded sets. Interval analysis began as a tool for bounding the rounding errors. The classical interval arithmetic appeared in 1960s [19] and was represented by the one-dimensional interval Moore's arithmetic approach. Interval methods find *all* solutions to a set of nonlinear equations in a given interval vector or a multi-dimensional box [23]. With the inherent ability of intervals

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to represent errors from all sources and to rigorously propagate their interactions, the validity of answers from the most extensive computations can now be guaranteed.

If this interval is used in the computation, the dependence can cause the widening of the final interval bound on the function value. For example, when doing the Gaussian elimination to solve the system of linear equations, dependence can cause a catastrophic numerical instability (especially in large-scale problems), which is exposed by the widening of resulting intervals. The width growing of resulting intervals is fully understandable intuitively, and in the subject literature it is called *the principle of increasing entropy* [20,21]. These problems can be relaxed using the *dependent interval arithmetic operations*, usually applied to interval subtraction [22,23]. In the above rules of interval arithmetic, the division by an interval containing zero must be excluded. The resulting arithmetic is called the *extended interval arithmetic* [23]. We note that the problem of determining the hull of the solution set of an arbitrary (non-degenerate) linear system is *NP*-hard [23], where different approximate algorithms are proposed in [22,23].

The interval mathematics is necessary in various new science branches, as for example, in artificial intelligence. An important area of artificial intelligence is the fuzzy set theory (the literature on this topic is rich and we only list references that are directly relevant for our development [9–12]). The introduction of ***fuzzy sets*** in interval mathematics is linked with the encoding and processing forms of information that are more sophisticated than numbers or symbols, namely pieces of information provided by human beings. A fuzzy set (mathematical quantity) is described by the membership function that has the shape of probability distribution functions. ***Closed intervals*** are generalized by fuzzy intervals with upper semi-continuous membership functions, i.e., by definition, whose level-cuts [α-cuts, where $\alpha \in (0;1)$] are closed intervals. ***A fuzzy interval*** is a fuzzy set in the real line whose level-cuts are intervals. A fuzzy interval is a convenient representation of imprecisely specified ranges of variables, conveying more information (for example, degree of probability) than a usual crisp interval. Finally, we note that, for many years, professor Zadeh (a pioneer in fuzzy set researches) has been claiming in his publications that both approaches (interval and fuzzy) are not contradictory but rather complementary (for example, see Ref. [24]).

In [20] the several examples of complete and partial correlations are provided. The consideration of the correlation of variables is very important, since it is a way to reduce the extension (conservativeness) of resultant intervals, which are less conservative than the results of classical interval arithmetic. The consideration of correlation is explained in [25] as well. Additionally, in this reference the affine interval arithmetic is explained. In [20,21] the interval arithmetic operations with correlated interval variables, based on Relative-Distance-Measure, are proposed. Ref. [26] explains the interval arithmetic operation in three characteristic cases: (1) not correlated variables, (2) high positive correlation among variables, and (3) high negative correlation among variables.

Note that the correlation of input variables is usually a problem-dependent. In this paper, for the consideration of input power flow based variable uncertainties, the non-conservative interval arithmetic operations are proposed. Additionally, the correlation between input variables is analyzed, where the correlation is defined by the degree of correlation (which can vary from -1 to 1). Therefore, as a result, narrowed resulting interval variables are calculated and compared with the ones obtained from the classical interval (Moore's) arithmetic. Later, the proposed interval arithmetic operations with correlated variables, which reduce the calculated uncertainty conservativeness, are directly included in the distribution power flow algorithm. A special attention is paid to the uncertainty and correlation of loads and DGs, which are here jointly named as *shunts*.

The main contributions of the paper can be summarized as:

(1) Correlations of main input variables (DGs and loads) are investigated and mathematically specified by the degree of correlation of interval variables.
(2) Population of the correlation matrix for main input variables (DGs and loads).
(3) Uncertainties and correlations of DGs and loads (represented by the proposed interval arithmetic) are directly included (by the correlation matrix) to the proposed modified forward-backward distribution power flow algorithm.
(4) Conservativeness is reduced significantly and therefore the calculation results are more realistic.
(5) Obtained results are verified by the combined Monte Carlo selection and power flow calculations.

This paper is organized in six sections. After the Introduction section, Section 2 explains the mathematical background of the proposed interval approach. Section 3 gives a modified algorithm of distribution power flow calculations, which considers the uncertainty of variables and its correlation. Section 4 explains the calculation of input correlations based on historically recorded input patterns. Following that, the calculation results obtained by the proposed procedure are provided in Section 5. The conclusions are given in Section 6.

2. Mathematical background

A (real) interval is a set of the form:

$$x \equiv [\underline{x}, \bar{x}] := \{x \in \boldsymbol{R} \quad |\underline{x} \leqslant x \leqslant \bar{x}\}, \tag{1a}$$

where $\underline{x}$, $\bar{x}$ are elements of $\boldsymbol{R}$ and represents the lower and upper interval bounds (endpoints), respectively. In particular, intervals are closed and bounded subsets of $\boldsymbol{R}$.

Using the concept of Relative-Distance-Measure variables, this interval can be represented in the following way [20,21]:

$$x \in [x_m + \alpha_x \Delta L_x], \quad \alpha_x \in [-1, 1]; \quad x \in [x_m - \Delta L_x, x_m + \Delta L_x], \tag{1b}$$

where

$$x_m = \frac{\bar{x} + \underline{x}}{2}; \tag{1c}$$

$$\Delta L_x = \bar{x} - x_m = x_m - \underline{x}; \tag{1d}$$

x_m – interval middle (central) point;
α_x – real number, with values between -1 and 1;
ΔL_x – half of interval width.

Eq. (1) are graphically illustrated in Fig. 1. Introducing the variable α_x, the interior of the interval $[\underline{x}, \bar{x}]$ becomes not anonymous, achieves a meaning and can be a part of following calculations.

2.1. Basic interval operations

The basic arithmetic operations with interval variables a and b used in this paper are [20,21]:

Addition

$$c = [\underline{c}, \bar{c}] = a + b = a_m + \alpha_a \Delta L_a + b_m + \alpha_b \Delta L_b, \quad \alpha_a, \alpha_b \in [-1, 1]. \tag{2}$$

Subtraction[1]

$$c = [\underline{c}, \bar{c}] = a - b = a_m + \alpha_a \Delta L_a - b_m - \alpha_b \Delta L_b, \quad \alpha_a, \alpha_b \in [-1, 1]. \tag{3}$$

[1] Note that by subtraction of two equal intervals by using the classical (Moore's) interval arithmetic [19], a resulting interval is double in width – the principle of increasing entropy [20] {for example, [1,2] – [1,2] = [–1,1]}. On the other hand, by application of (3) the zero interval is obtained, which is significantly less conservative compared to the classical interval arithmetic [19].