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# A bi-objective DC-optimal power flow model using linear relaxation-based second order cone programming and its Pareto Frontier

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ABSTRACT

The DC optimal power flow (DC-OPF) plays an important role in the operation and planning of modern power systems. In this paper, a bi-objective DC-OPF model minimizing both network losses and generation costs is introduced, which can further be converted into a single objective model via the weighted sum method. Furthermore, the Pareto Frontier is employed to solve this problem. In the mathematical view, the model is a special non-convex quadratic constraints quadratic programming problem. In order to obtain a continuous Pareto Frontier, the original non-convex feasible region is relaxed to its convex hull using a linear relaxation-based second order cone programming method. Compared with the semi-definite relaxation method, the proposed method can greatly reduce the number of dummy variables and the complexity of solutions. Finally, simulations on eight small systems and four practical, large systems are performed, in addition to the comparison of a Monte Carlo simulation. The results verify the effectiveness of the proposed algorithm.



## 1. Introduction

With different optimization objectives, the optimal power flow (OPF) can be used to schedule the power output of each unit in order to achieve a better operation state while satisfying the real-time power balance within the transmission capacity of the power network [1]. Generally, the OPF models include AC optimal power flow (AC-OPF) and DC optimal power flow (DC-OPF) models [2]. Mathematically, the AC-OPF is a typically nonlinear and non-convex optimization problem with certain constraints in nature [3]. Many optimization methods have been applied to solve the OPF models, such as Newton method, artificial intelligence, active-set, trust-region, and modern interior point algorithm [4–7]. Comparing to AC-OPF, DC-OPF is a linearized model without the consideration of the voltage magnitude and reactive power that is commonly used in the nowadays practical industrial power market clearing [8]. The common objectives of DC-OPF include the economy [9,10], energy-saving, emission minimization [11–13], and so on. Moreover, the DC-OPF with the consideration of prohibited zones, wind power uncertainties, flexible transmission line impedance and phase shifting transformers is studied in [14–18].

However, the traditional DC-OPF model usually ignores the network losses and is simplified as a convex programming problem, which may lead to a large gap between the original AC-OPF. Additionally, the interconnected large-scale systems over long distance transmission have been widely adopted in China due to the uneven distribution of load and energy resources. Under such circumstances, the network losses have significant effects on unit scheduling and economic optimization. Therefore, the consideration of network losses in the DC-OPF models will make the solution more realistic.

In [19], a multi-objective OPF model was established to minimize the fuel cost, power losses, and total real power output of all generators. The model was solved by a non-linear interior-point OPF algorithm based on power current hybrid mismatch formulations in rectangular coordinates. In [20–23], different artificial-intelligence algorithms were applied to solve the OPF models considering the network losses. However, the results of these methods are highly dependent on the populations in the algorithms, and the computation is time-consuming compared with the deterministic algorithms. Ref. [24] studied a dynamic linear DC-OPF model, iteratively searching the optimal solution after approximating network losses as linear cut-sets. Ref. [25] built a new fictitious nodal demand (FND) model to adjust the offset of network losses. Ref. [26] proposed a non-linear OPF algorithm to solve the optimality conditions using a pure Newton-Raphson solution procedure.

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The methods above obtained promising results; however, rather than global, only a local optimal solution could be achieved.

Ref. [27] built a multi-objective model to minimize both the generation cost and the network losses, in which an approximate solution was obtained by the proposed semi-definite relaxation method. However, the method brings a heavy computational burden to large-scale systems because it needs to introduce $n^2$ new dummy variables, where $n$ is the number of the optimization variables. Therefore, a method of linear relaxation and second order cone constraints is proposed to solve the DC-OPF model in this work, which could reduce a large number of dummy variables and significantly improve the computing efficiency while obtaining almost the same global optimal solution with the semi-definite relaxation method. Compared with the semi-definite relaxation method, the proposed method is more applicable for a large-scale system optimization.

Furthermore, the Pareto Frontier of the proposed multi-objectives model can be achieved by the weighted sum method. Interestingly, the Pareto Frontier takes on a similar hyperbolic functional feature under different load levels. Finally, the proposed method can be easily extended to a multi-stage dynamic DC-OPF.

## 2. Mathematical formulation of bi-objective DC-OPF

The main objectives of the OPF aim to minimize the total generation costs of the system as well as reduce the network losses while meeting the power balance constraints, output limits of the units, and restrictions of branch power. The generation costs function is commonly a quadratic function of the generation. As shown in (1), $f_1$ and $f_2$ indicate the total generation cost and the total network loss respectively.

$$\min_{\boldsymbol{P}_g} \quad f_1(\boldsymbol{P}_g) + f_2(\boldsymbol{P}_g) \tag{1-a}$$

$$f_1(\boldsymbol{P}_g) = \sum_{i=1}^{N_g} (a_i P_{g,i}^2 + b_i P_{g,i} + c_i) \tag{1-b}$$

$$f_2(\boldsymbol{P}_g) = P_{loss} \tag{1-c}$$

where ($a_i$, $b_i$, $c_i$) are the triplet coefficients of the quadratic cost function; $N_g$ is the number of conventional thermal units in the system; $P_g$ is the generation of the $i$th thermal unit, and $P_{loss}$ is the total network losses.

The constraints of DC-OPF can be cast as follows:

(i) **Energy balance constraint**

$$\sum_{i=1}^{N_g} P_{g,i} = \sum_{j=1}^{N_d} P_{d,j} + P_{loss} \tag{2}$$

where $P_{d,j}(t)$ is the demand of load $j$, and $N_d$ is the number of load.

(ii) **The output power restrictions of thermal units**

$$P_{g,i}^{\min} \leqslant P_{g,i} \leqslant P_{g,i}^{\max} \tag{3}$$

where $P_{g,i}^{\min}$ and $P_{g,i}^{\max}$ are the upper and lower limits of the $i$th thermal unit's generation, respectively.

(iii) **Security operation constraints of the grid**

$$-P_l^{\max} \leqslant P_l \leqslant P_l^{\max}, \quad l = 1, 2, \ldots, N_l \tag{4}$$

where $P_l^{\max}$ denotes the maximum transmission power of branch $l$ (from bus $i$ to bus $j$), and $N_l$ is the number of the branches.

According to [28], the transmission power can be reformulated to the node power injection via the network shift distribution factor, yielding

$$-P_l^{\max} \leqslant \sum_{i=1}^{N_g} G_{l,i} P_{g,i} - \sum_{j=1}^{N_d} H_{l,j} P_{d,j} \leqslant P_l^{\max} \tag{5}$$

where $G_{l,k}$ and $H_{l,k}$ are network shift distribution factors, indicating the sensitivity information of the branch power with respect to the power injection to generator and load nodes, respectively. It should be noted that the average transmission power is employed in (5), since the transmission power is different at the two ends of the transmission line with the consideration of network losses.

As for the network losses, the assumption is rationally made that the voltage amplitudes of all nodes approximate to 1.0 p.u. because the OPF model is based on the DC power flow. Thus, the total network losses can be given as (6).

$$P_{loss} = \sum_{l=1}^{N_l} I_l^2 R_l \approx \sum_{l=1}^{N_l} P_l^2 R_l \tag{6}$$

where $I_l$, $R_l$, $P_l$ are the current impedance and average power of branch $l$.

We may recall the transmission power $P_l$ defined in (5) that is expressed as the network transfer distribution factors. The total network losses can be further given by

$$P_{loss} = \sum_{l=1}^{N_l} \left( \left( \sum_{i=1}^{N_g} G_{l,i} P_i - \sum_{j=1}^{N_d} H_{l,j} P_{d,j} \right)^2 R_l \right) \tag{7}$$

In matrix forms, the objective function and its constraints are given as follows:

**Objective function:**

$$\min_{\boldsymbol{P}_g} f_1(\boldsymbol{P}_g) + f_2(\boldsymbol{P}_g) \tag{8 - a}$$

$$f_1(\boldsymbol{P}_g) = \boldsymbol{P}_g^T \boldsymbol{A} \boldsymbol{P}_g + \boldsymbol{b}^T \boldsymbol{P}_g + c$$

$$\begin{aligned} f_2(\boldsymbol{P}_g) &= P_{loss} = (\boldsymbol{G}_1 \boldsymbol{P}_g - \boldsymbol{G}_2 \boldsymbol{P}_d)^T \boldsymbol{R} (\boldsymbol{G}_1 \boldsymbol{P}_g - \boldsymbol{G}_2 \boldsymbol{P}_d) \\ &= \boldsymbol{P}_g^T \boldsymbol{G}_1^T \boldsymbol{R} \boldsymbol{G}_1 \boldsymbol{P}_g - 2(\boldsymbol{P}_d^T \boldsymbol{G}_2^T \boldsymbol{R} \boldsymbol{G}_1) \boldsymbol{P}_g + \boldsymbol{P}_d^T \boldsymbol{G}_2^T \boldsymbol{R} \boldsymbol{G}_2 \boldsymbol{P}_d \end{aligned}$$

where $\boldsymbol{R} = diag(R_l)_{N_l \times N_l}$ is the impedance matrix; $\boldsymbol{G}_1 = (G_{l,i})_{N_l \times N_g}$ and $\boldsymbol{G}_2 = (H_{l,j})_{N_l \times N_d}$ are the transfer distribution factor matrices; and $\boldsymbol{A} = diag(a_i)_{N_g \times N_g}$, $\boldsymbol{b} = diag(b_i)_{N_g \times 1}$, $c$ are the coefficients of the quadratic cost function.

**Constraints:**

$$\boldsymbol{e}_1^T \boldsymbol{P}_g - \boldsymbol{e}_2^T \boldsymbol{P}_d - P_{loss} = 0 \tag{8-b}$$

where $\boldsymbol{e}_1$ is the unit vector of $N_g \times 1$ and $\boldsymbol{e}_2$ is the unit vector of $N_d \times 1$.

Substitute the losses expression (8-a) into (8-b), and we have

$$\boldsymbol{P}_g^T \boldsymbol{G}_1^T \boldsymbol{R} \boldsymbol{G}_1 \boldsymbol{P}_g - 2(\boldsymbol{G}_1^T \boldsymbol{R} \boldsymbol{G}_2 \boldsymbol{P}_d + \boldsymbol{e}_1)^T \boldsymbol{P}_g + \boldsymbol{P}_d^T \boldsymbol{G}_2^T \boldsymbol{R} \boldsymbol{G}_2 \boldsymbol{P}_d + \boldsymbol{e}_2^T \boldsymbol{P}_d = 0 \tag{8-b$'$}$$

Here, we define three matrices for simplification, such that $d_0 = \boldsymbol{P}_d^T \boldsymbol{G}_2^T \boldsymbol{R} \boldsymbol{G}_2 \boldsymbol{P}_d + \boldsymbol{e}_2^T \boldsymbol{P}_d$, $\boldsymbol{\eta} = \boldsymbol{G}_1^T \boldsymbol{R} \boldsymbol{G}_2 \boldsymbol{P}_d + \boldsymbol{e}_1$ and $\mathbf{X} = \boldsymbol{G}_1^T \boldsymbol{R} \boldsymbol{G}_1$. Therefore, the energy balance equation of the system can be recast as

$$\boldsymbol{P}_g^T \boldsymbol{X} \boldsymbol{P}_g + \boldsymbol{\eta}^T \boldsymbol{P}_g + d_0 = 0 \tag{8-b$''$}$$

$$-\boldsymbol{P}_l^{\max} \leqslant \boldsymbol{G}_1 \boldsymbol{P}_g - \boldsymbol{G}_2 \boldsymbol{P}_d \leqslant \boldsymbol{P}_l^{\max} \tag{8-c}$$

$$\boldsymbol{P}_g^{\min} \leqslant \boldsymbol{P}_g \leqslant \boldsymbol{P}_g^{\max} \tag{8-d}$$

Combining (8-a), (8-b''), (8-c), and (8-d), the matrix form of the multi-objective DC-OPF can be finally formulated as (9).