Electrical Power and Energy Systems 86 (2017) 71–80

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A recursive power flow method for radial distribution networks: Analysis, solvability and convergence

M.R. Shakarami [a,*], H. Beiranvand [a], A. Beiranvand [b], E. Sharifipour [c]

[a] Department of Electrical Engineering, Lorestan University, Khorramabad, Iran
[b] Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran
[c] Lorestan Electrical Power Distribution Company, Khorramabad, Iran

ARTICLE INFO

Article history:
Received 28 May 2016
Received in revised form 4 August 2016
Accepted 6 October 2016

Keywords:
Load flow
Graph theory
Power distribution systems
Convergence

ABSTRACT

A new Recursive Power Flow (RPF) algorithm for balanced/unbalanced radial distribution systems is proposed in this paper. Four generalized constant matrixes, that are calculated based on the graph theory, are used to obtain the power flow solution. These matrixes completely describe topology of a distribution system and models of its elements. In this method, each branch of a distribution system is considered as a bipolar. The method is able to consider detailed model of the elements. Solvability and convergence properties of the method have been mathematically presented. The method has been studied on the standard 16, 33 and 69-bus balanced distribution systems. In addition, this method is tested on 4 and 13-bus unbalanced standard distribution systems. Also, the method is tested on a 1337-bus real distribution system in Lorestan Electrical Power Distribution Company, Iran. The simulation results demonstrate the accuracy, computational efficiency and robustness of the proposed method.

1. Introduction

Generally, power flow is essential in any program for analyze or optimization of distribution networks. Today, load flow methods with high accuracy, robustness and low computing burden, while considering full model of the elements of the distribution networks, are of interest.

Mainly, published power flow methods in the literature for distribution networks can be classified as follows:

- The Z-bus Power Flow (ZPF) methods [1,2].
- The Newton-Raphson Power Flow (NRPF) methods [3–8].
- The Backward/Forward Sweep Power Flow (BFSPF) methods [9–12].
- Optimization-Based Power Flow (OBPF) methods [13,14].
- Direct Lower Flow (DLF) methods [15–17].

The Gauss implicit Z-matrix method in [2] uses lower upper (LU) factorization in its main loop and does not clearly used for a radial distribution system. Ill-conditioning problems may cause NRPF methods to fail due to radial structure, large number of buses and branches, and high R/X ratio [17,18].

The BFSPF methods have the most development in the recent years, because they are simple to implement, and are able to consider the detailed models of the distribution network elements. Different versions of the BFSPF have been reported in the literature. For instance, the Fortescue coordinates was proposed instead of phase coordinates in [19]. In [20], an improved BFSPF for radial distribution systems was presented.

The OBPF methods formulate the power flow as an optimization problem and solve it by optimization tools. These methods depend on mathematical optimization toolboxes. In these methods, accuracy and execution time depend on the utilized optimization method.

The DLF method, presented in [17], removes backward/forward sweeps of the BFSPF, the time-consuming LU decomposition for Z-bus matrix calculations in the ZPF, forward/backward substitution of the Jacobian matrix or the admittance matrix, required in the traditional load flow methods. However, this method assumes some simplification to calculate the developed matrixes for a distribution system. In the DLF method, three-phase transformer modeling was not considered. Also, in this method parallel sections of transmission lines could not be directly modeled. Three-phase transformer winding connections introduce some challenges for power flow algorithms. Some transformer configurations such as Delta-Grounded Wye, with any vector group, produce a singularity in their admittance matrixes. This makes impedance matrix calculation to be impossible. While, impedance matrix is necessary in

* Corresponding author.
E-mail addresses: shakarami.mr@lu.ac.ir (M.R. Shakarami), beiranvand.ha@fe.lu.ac.ir (H. Beiranvand), a_beiranvand@tabrizu.ac.ir (A. Beiranvand), sharifipour_322@yahoo.com (E. Sharifipour).

http://dx.doi.org/10.1016/j.ijepes.2016.10.002
0142-0615/

such methods in [21]. Ignoring the detailed models of the elements, especially transformers model, may adversely influence the results of the load flow solutions in distribution systems.

This paper proposes a new Recursive Power Flow (RPF) algorithm for radial distribution networks to overcome the DLF method drawbacks. In the proposed method, four generalized constant matrixes are presented. These matrixes completely describe the distribution system topology and the element models. In RPF method, an iterative recursive formulation to calculate voltage of buses has been proposed using the generalized constant matrixes and power injections. This method does not need optimization algorithms, LU factorization and Jacobian matrix calculations at each iteration. The RPF method reduces backward/forward loops in BFSPF methods and finds the power flow solution directly. The RPF method performance is investigated using 16-bus, 33-bus and 69-bus balanced standard test systems. In addition, a simple unbalanced 4-bus test system, an IEEE 13-bus distribution system and a real large-scale distribution system in Lorestan Electrical Power Distribution Company, Iran, has been analyzed by this method. Results validate the effectiveness of the proposed method.

The paper is organized as follows: Section 2 presents the components models that are used to compute the power flow solutions of distribution systems. Section 3 describes the general formulation of the RPF method. Calculation procedure for generalized matrixes using graph theory is given in Section 4. Section 5 evaluates the PV buses in the RRF method. In Section 6, a comparison between the proposed method and the DLF method in [17], theoretically and numerically, has been shown. Solvability and convergence properties of the proposed method are proven in Section 7. The simulation results for several case studies are given in Section 8. Lastly, conclusions are presented in Section 9.

2. Model of elements

The modeling of distribution branches (especially, line segments and transformers) is a vital step in the power flow analysis of a distribution feeder. In this paper, a unified bipolar model for each branch, as shown in Fig. 1(a), is considered. Voltage and current equations related to nodes 1 and 2 are as follows:

$$\begin{aligned} \mathbf{V}_2 &= \mathbf{A}\mathbf{V}_1 + \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 + \mathbf{D}\mathbf{I}_2 \end{aligned} \tag{1}$$

Bipolar matrixes, for every black-box element in a distribution system, can be obtained by open circuit and short circuit measurements. For example, from secondary open circuit test $\mathbf{A} = \mathbf{V}_2/\mathbf{V}_1$ and $\mathbf{C} = \mathbf{I}_1/\mathbf{V}_2$.

2.1. Line models

To take into account the unbalanced aspects, it is important to consider each phase, separately. In this case, the correct spacing between conductors is needed. The self and mutual impedances for different phases can be determined by Carson's technique [22], then the impedances of the line segments and admittance matrixes, as shown in Fig. 1(b), can be calculated. The bipolar parameters of Fig. 1(b) can be computed as Eq. (2). In this equation $\mathbf{U}_3$ is a 3×3 identity matrix.

$$\begin{aligned} \mathbf{A} &= \left(\mathbf{U}_3 + \tfrac{1}{2}\mathbf{Z}_{abc}\mathbf{Y}_{abc}\right)^{-1} \\ \mathbf{B} &= -\left(\mathbf{U}_3 + \tfrac{1}{2}\mathbf{Z}_{abc}\mathbf{Y}_{abc}\right)^{-1}\mathbf{Z}_{abc} \\ \mathbf{C} &= \mathbf{Y}_{abc} + \tfrac{1}{4}\mathbf{Y}_{abc}\mathbf{Z}_{abc}\mathbf{Y}_{abc} \\ \mathbf{D} &= \mathbf{U}_3 + \tfrac{1}{2}\mathbf{Z}_{abc}\mathbf{Y}_{abc} \end{aligned} \tag{2}$$

where $\mathbf{Z}_{abc}$ and $\mathbf{Y}_{abc}$ are impedance and admittance matrixes of a line section, respectively.

2.2. Three-phase distribution transformers

Typically, the bipolar parameters for a three-phase Dyn5 distribution transformer are calculated using the method described in [22] is as follows:

$$\begin{aligned} \mathbf{A} &= \frac{1}{n_t}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \\ \mathbf{B} &= -\begin{bmatrix} Zt_a & 0 & 0 \\ 0 & Zt_b & 0 \\ 0 & 0 & Zt_c \end{bmatrix} \\ \mathbf{C} &= \frac{1}{n_t}\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \\ \mathbf{D} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{3}$$

where $n_t = \sqrt{3}$ for per unit calculations and Z_t is per unit short circuit impedance of each phase winding.

2.3. Three-phase distribution transformers

A voltage regulation transformer (VRT) [22] comprises an autotransformer and a load tap changing mechanism. The VRT is able to regulate the secondary voltage of the autotransformer from -10% up to 10% of the nominal secondary output voltage, usually in 32 steps. The VRT can be modeled like a distribution transformer in the power flow algorithm by defining its bipolar matrixes. For example, bipolar matrixes for an ideal VRT with Wye-Wye connection are given as follows:

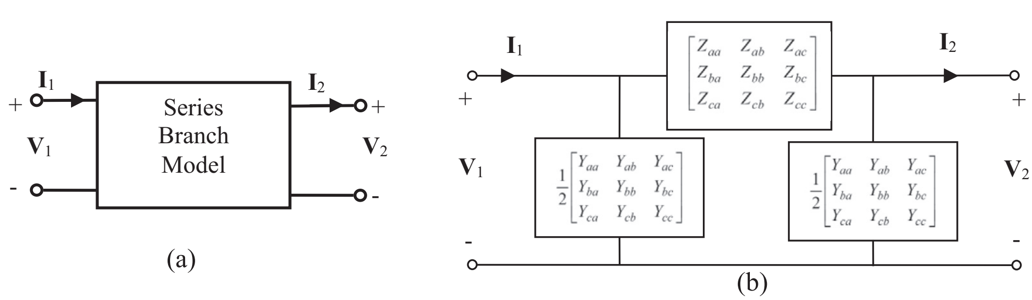

Fig. 1. Branch model, (a) bipolar model of a series element and (b) the π-equivalent model of a three-phase line segment.