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# Dam structural behavior identification and prediction by using variable dimension fractal model and iterated function system

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ABSTRACT

According to the observations of dam structural health monitoring, iterated function system is adopted to implement the analysis and forecast for dam structural behavior. Firstly, the multifractal detrended fluctuation analysis (MF-DFA) method is employed to identify the fractal characteristics in the measured data series of dam structural behavior. Secondly, the iterated function system algorithm is studied to build the fitting model. The ways to determine the interpolating points (position and number) and vertical scaling factors are given in detail. Thirdly, the variable dimension fractal model and iterated function system are combined to forecast the dam structural behavior. Lastly, the displacement behavior of one concrete gravity dam is analyzed and predicted by the proposed approach. It is shown that the whole trend and detail characteristics of dam structural behavior observed can be described well, and the prediction precision can be improved.



## 1. Introduction

During dam service, many factors comprehensively affect dam structural behavior (such as deformation, stress and strain, seepage, etc.). The complicated and time-varying correlation among the factors causes the strong nonlinear characteristics of measured data series on dam structural behavior [1–3]. According to the observations of dam structural behavior, some mathematical methods are often used to analyze and identify above nonlinear characteristics, and build the fitting and forecasting model of dam structural behavior. Dam safety can be monitored with the built model [4–6].

Statistical models based on monitoring data have been used for decades for this purpose [6–8]. In particular, the hydrostatic-season-time method is fully implemented in engineering practice, although some limitations have been pointed out. In other fields of science, powerful tools such as neural networks and support vector machines have been developed, which make use of observed data for interpreting complex systems [2,9–11]. The limitations of traditional statistical tools and the availability of these advanced learning algorithms have motivated dam engineers to search the possibilities of the latter for building dam behavior models, as well as for analyzing dam behavior.

However, the above mentioned methods cannot identify long-term correlate behavior. In the fractal theory, fractal is regarded as a basic attribute of nature. The whole and its part are self-similar. The whole fractal dimension obtained can be applied to the changing trend forecast [12–14]. In a previous paper, monofractal exponents were obtained based on observed time series in order to give information on the inherent evolution law of a dam system [15]. The investigated example indicates that dam structure has self-similarity characteristics. In recent years the detrended fluctuation analysis (DFA) method has become a widely used technique for the determination of monofractal scaling properties and the detection of long-range correlations in noisy and nonstationary time series. It has successfully been applied to various fields such as long-time weather records, cloud structures, geology, and solid state physics. Multifractals can be decomposed into many-possibly infinitely many sub-sets characterized by different scaling exponents. In another previous paper, multifractal DFA (MF-DFA) was employed to reveal the time scale effect and the nonlinear dynamic evolution law of dams [16]. The results show that this method can reliably determine the multifractal scaling behavior of time series

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of existing dams. The fractal theory can be applied to predict and diagnose dam behavior.

In the paper, the MF-DFA is used to identify the multifractal characteristics of observed dam structural behavior. The iterated function system (IFS) is adopted to fit the time curve on observed data of dam structural behavior. The ways to improve the interpolation accuracy are also studied. Due to weak extension capability of above fractal interpolation method, the variable dimension fractal model is combined with IFS to implement the dam structural behavior forecast. The proposed method is applied to an actual dam. The nonlinear and time-varying characteristics of dam displacement are analyzed. The effect fitting and forecasting dam displacement is appraised.

## 2. MF-DFA for measured data series of dam structural behavior

Based on the previous DFA, a robust multifractal analysis namely MF-DFA is proposed [17,18]. MF-DFA method takes fluctuant average of time series in each partition interval as statistical points and determines generalized Hurst exponent depending on power-law property of fluctuation function to measure stationary and non-stationary sequence structure and fluctuation singularity. The advantages of this method are that it can find the long-range correlations of non-stationary time series. And It is demonstrated with the computer simulation that the effect using MF-DFA method to analyze multifractality for non-stationary time series was the best in all methods [19].

The concrete steps of analyzing measured data characteristics of dam's service behavior based on MF-DFA are as follows:

(1) Cumulative deviation of time series $\{x_t, t = 1, 2, \ldots, n\}$ of dam's prototype monitoring data is calculated as

$$Y(i) = \sum_{t=1}^{i} (x_t - \bar{x}) \tag{1}$$

where $\bar{x} = \frac{1}{N}\sum_{t=1}^{i} x_t$.

(2) Divide sequence $Y(i)$ into $m$ non-overlapping intervals $v$. Each interval contains the same number of points $s$, where integral part is $m = [N/s]$. Since the length of the sequence is often not an integral multiple of $s$. In order not to produce surplus, the same procedure is repeated from the opposite end from the $m+1$-th interval. Thereby, $2m$ intervals are obtained altogether.

(3) Fitting polynomial of the $v$-th interval through a least-square fit of the data for each interval $v (v = 1, 2, \ldots, 2m)$ can be got as:

$$\hat{y}_v(i) = \hat{a}_0 + \hat{a}_1 i + \cdots + \hat{a}_k i^k, \quad i = 1, 2, \cdots, s, \quad k = 1, 2, \cdots \tag{2}$$

$Y_s(i)$ which the time series removing the trend is denoted by shows the difference between the original series and fitted values.

$$Y_s(i) = Y(i) - \hat{y}_v(i) \tag{3}$$

where $\hat{y}_v(i)$, called $k$-order MF-DFA, is the local trend function of the $v$-th interval. $k$ is the different fitting order. In MF-DFA$k$ ($k$th order MF-DFA) trends of order $k$ in the profile (or, equivalently, of order $k-1$ in the original series) are eliminated.

(4) Calculate the variance of each interval which has been removed the trend.

If $v = 1, 2, \ldots, m$,

$$F^2(v, s) = \frac{1}{s}\sum_{i=1}^{s} Y_s^2[i] = \frac{1}{s}\sum_{i=1}^{s} \left(y((v-1)s + i) - \hat{y}_v(i)\right)^2 \tag{4}$$

If $v = m+1, m+2, \ldots, 2m$,

$$F^2(v, s) = \frac{1}{s}\sum_{i=1}^{s} Y_s^2[i] = \frac{1}{s}\sum_{i=1}^{s} \left(y((n-(v-1))s + i) - \hat{y}_v(i)\right)^2 \tag{5}$$

Obviously, $F^2(v, s)$ is concerned with the fitting order. Different orders have different abilities to eliminate the trend.

(5) Average and extract a root for all variances of equal-length intervals. Then the $q$-order fluctuation function of the whole sequence can be obtained:

$$F_q(s) = \left\{ \frac{1}{2m}\sum_{v=1}^{2m} \left[F^2(v, s)\right]^{q/2} \right\}^{1/q} \tag{6}$$

In general, the index variable $q$ can take any real value. For $q = 0$, the fluctuation function can be determined as below equality:

$$F_0(s) = \exp\left\{ \frac{1}{4m}\sum_{v=1}^{2m} \ln\left[F^2(v, s)\right] \right\} \tag{7}$$

For $q = 2$, it can be seen that Eqs. (14) and (4) are the same, the standard DFA procedure is retrieved. At this point, DFA is the special form of MF-DFA.

For positive $q$, the segments $v$ with large variance (i.e., large deviation from the corresponding fit) will dominate the average $F_q(s)$. Therefore, if $q$ is positive, $h(q)$ describes the scaling behavior of the segments with large fluctuations; and generally, large fluctuations are characterized by a smaller scaling exponent $h(q)$ for multifractal time series. For negative $q$, the segments $v$ with small variance will dominate the average $F_q(s)$. Thus, for negative $q$ values, the scaling exponent $h(q)$ describes the scaling behavior of segments with small fluctuations, usually characterized by a larger scaling exponents.

Therefore, different $q$ values have different effects on fluctuation functions.

(6) Determine the scaling exponent of fluctuation function. Varying the value of $s$ in the range from $s_{\min} \approx 5$ to $s_{\max} \approx N/4$, and repeating the procedure described above for various scales $s$, $F_q(s)$ will increase with increasing $s$. Then analyzing log–log plots $F_q(s)$ vs. $s$ for each value of $q$, the scaling behavior of the fluctuation functions can be determined. If the series $x_i$ is long-range power-law correlated, $F_q(s)$ increases for large values of $s$ as a power-law

$$F_q(s) \sim s^{h(q)} \tag{8}$$

In general the exponent $h(q)$ will depend on $q$. For stationary time series, $h(2)$ is the well defined Hurst exponent $H$. Thus, $h(q)$ is called the generalized Hurst exponent. Monofractal time series are characterized by $h(q)$ independent of $q$. The different scaling of small and large fluctuations will yield a significant dependence of $h(q)$ on $q$.

The above equality can be also expressed as $F_q(s) = As^{h(q)}$. Take logarithm for the both sides of the equality

$$\ln\left(F_q(s)\right) = \ln A + H(q)\ln(s) \tag{9}$$

A corresponding fluctuation function value $F_q(s)$ can be obtained for each partition length $s$; different $F_q(s)$ can be got by using different constant $s$. By using the least square method to make linear regression for the above equality, slope estimated value obtained is $q$-order generalized Hurst exponent $h(q)$.

Generalized Hurst exponent $h(q)$ has the significance of scaling exponent of DFA, but $h(q)$ is concerned with $q$. Time series is monofractal if $h(q)$ has nothing with $q$ and time series is multifractal if $h(q)$ is a function of $q$.