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Fuzzy heuristic solution approaches for the warm/cold lot sizing problem

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ABSTRACT

In this paper we introduce fuzzy versions some rule based lot sizing heuristics for the dynamic lot-sizing problem with warm/cold process. In our setting "the demand at each period" and "the warm system threshold" (production/order quantity required for keeping the system warm on to next period) are fuzzy numbers. Similar to the crisp counterpart setting of the problem, horizon length, production capacity at each period, inventory carrying cost and warming cost are the parameters with crisp values. The objective is to find the cost minimizing production scheme throughout the horizon. The rule based fuzzy heuristics we introduce are: "fuzzy silver meal algorithm", "fuzzy part period algorithm", and "fuzzy least unit cost algorithm". We illustrate implementation of proposed heuristics through examples. In a numerical study we present comparison results of heuristics based on various performance criteria.

1. Introduction

There is an extensive literature built up on the dynamic lot-sizing problem after its first introduction by Wagner and Whitin [58]. This problem is simply defined as the determination of the cost minimizing production scheme over a planning horizon for a storable item facing known demands, when there is no capacity on the production quantity and shortages are not allowed. We will refer to this setting, by Wagner and Whitin [58], as the "classical problem" in the rest of this paper. Structural properties of the optimal production scheme and an optimal solution algorithm for the classical problem are provided by Wagner and Whitin [58]. Structural properties indicate that forward solution algorithms are possible, and existence of planning horizons facilitates the development of algorithms with less complexity. We refer the reader to Buschkühl et al. [9] for a detailed review of the dynamic lot sizing problem literature and to Beck et al. [6] for a recent paper on improvements of dynamic lot sizing heuristics.

In the classical setting of the dynamic lot sizing problem, by Wagner and Whitin [58], demand at each period is variable but known. The objective is the determination of the production/ordering scheme for the planning horizon, specifically when to produce and how much to produce. The first feasible solution one can come up with for this problem is producing the demand of a period at its own period. Which yields no holding cost but maximum setup cost since production is done at every period and setup cost would be incurred for all the periods with positive demand. This solution can be improved by producing at earlier periods and carrying those items in stock until they are demanded, which yields decreased setup cost but increased holding cost. Solving this problem is complex unless some structural properties of the optimal solution are established. Wagner and Whitin [58], establishes these structural properties and propose a forward solution algorithm which yields the optimal solution in reasonable time. There are various generalizations of the classical problem in the literature. One generalization is the incorporation of "warm/cold process" setting proposed by Toy and Berk [53]. This generalization represents a real life situation where setup in a period may be carried on the next period under some circumstances. Each period in this problem setting has a fixed time interval. Under some production rate, if production quantity in a period is large enough to keep the production facility running there will be no need to shut down and restart the production facility, hence production will continue in the immediately next period without incurring any setup related costs. In this "warm/cold process" setting, the production process can be kept "warm" on to the next period depending on the production quantity. The production quantity is compared to a pre-defined quantity, referred as the *threshold*, such that if the production is at least as much as the threshold and there is production in the succeeding period, the process will be warm. The warm process results in

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Nomenclature

N	horizon length
$\tilde{D}_t$	fuzzy demand in period t, t = 1, 2,. ., N
R	production capacity in a period
$\tilde{Q}$	fuzzy warm system threshold in a period, in order to keep the process warm on to next period
$\tilde{E}_t$	fuzzy economic bound in period t
c	unit production cost
h	inventory holding cost (per unit per period)
ω	warming cost (per unit)
K	setup cost
x_t	production quantity in period t, t = 1, 2,. ., N
$\tilde{y}_t$	fuzzy on hand stock at the end of period t
z_t	cold system indicator (0: warm system; 1: cold system)

maintaining the production setup, hence, requiring no (or a minor) setup in the next period. However, producing less than the threshold level results in a "cold" process at the end of the period, and requires production to start anew with a major setup. We refer the reader to Toy and Berk [54] and Toy and Berk [53] for a broader discussion of the motivational examples for the dynamic lot-sizing problem in the presence of production quantity—dependent warm/cold processes. Alluded to their papers, warm/cold process is implemented in industries such as glass, steel and ceramic production where start-ups and shutdowns are costly. Likewise, processes in which the rate of the production can be changed in order to keep incessant production are in this category.

Even though the structural properties of the optimal solution are established and some solution algorithms are suggested for the classical problem, the complexity of obtaining the optimal solution initiated efforts on developing lot sizing heuristics based on simple stopping rules. Some examples for such rules are Silver–Meal [50], Part – Period Balancing [15], and Least Unit Cost [40]. Adaptations of existing heuristics and other original heuristics are proposed for extensions and generalizations of the classical problem, as well. For the warm/cold process dynamic lot-sizing problem Toy and Berk [54] have developed modified versions of traditional lot sizing heuristics, and provided comparison among suggested algorithms.

In the classical problem and in its generalizations, all parameters are assumed to be deterministic. However, as stated by Bushuev et al. [10], real-world involves a great deal of uncertainty; therefore, it is not realistic to assume certain parameters (i.e. demand, etc.) to be deterministic. As the length of the forecast horizon increases, uncertainty increases as well [41]. Salameh and Jaber [47] approve this remark by stating that some former studies consider defective rates in lot sizes as constants whereas other studies model them as random variables and the latter are more realistic. Similarly, De Bodt and Van Wassenhove [14] confirm this remark by addressing the demand uncertainty in lot sizing. Stochastic variables are generally used to deal with these uncertainties; for a review of stochastic lot sizing problems, we refer the readers to Sox et al. [51]. However, the uncertainty of the parameters can be determined by the nature of the problem at hand. For instance, when the case involves production-on-demand, demand becomes a deterministic parameter on which we have a priori knowledge [48]. On the contrary, Alonso-Ayuso et al. [2] state that demand is an uncertain parameter for an overwhelming majority of production plants. In fact, demand is the most obvious uncertainty involving parameter of a production system and considered uncertain by numerous researchers; some examples are [13,28,36,38,39,45,49,61,62].

Stochastic representation is not the only way of modeling uncertainty. Under some circumstances fuzzy set theory can be used conveniently. The reason for using fuzzy set theory which was introduced by Zadeh [64] is that it can deal with situations characterized by imprecision due to subjective and qualitative evaluations. Imprecision is represented by possibility rather than probability distributions [41]. Karwowski and Evans [25] listed three main reasons for incorporating fuzzy set theory in production management research as follows: (i) in the production management environment imprecision and vagueness are inherent to the decision maker's mental model of the problem under study, (ii) the information required to formulate a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable, (iii) imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information. The vagueness and imprecision in qualitative factors in production systems are generally characterized in linguistic or fuzzy terms [19,22,35,41]. Karsak and Tolga [24] declare that assessment of factors including flexibility, quality of the products, enhanced response to market demand, and reduction in inventory can neither be reflected by crisp values nor random processes. As a further example, Wong [60] considers unit holding and shortage costs being imprecise and claims that these parameters are best described with fuzzy sets theory because it transforms qualitative estimates into quantitative values.

The literature on fuzzy set theory and its applications are expanding considerably in the last decade. We refer the reader to the following work for detailed philosophy of fuzzy set theory and its applications: Prado et al. [43], Almulhim et al. [1], Mesiar and Stupnanova [37], Liu [32], Dubois and Prade [16], Magdalena et al. [34], Luhandjula [33], Kahraman et al. [23], Behounek and Hanikova [7], Tamir et al. [52]. Below we summarize the fuzzy set theory literature with particular emphasis on its application to lot sizing.

Fuzzy lot sizing heuristics have been a topic of interest for the researchers for the last two decades [21]. In terms of fuzzy heuristics, Lee et al. [30] examine the fuzzy part period balancing algorithm and Lee et al. [31] present a comparative study between the fuzzy part-period balancing, the fuzzy Wagner-Whitin and the fuzzy Silver-Meal methods. Callarman and Hamrin [11] deal with fuzzy economic order quantity (EOQ), fuzzy Wagner-Whitin and fuzzy part-period balancing algorithms under the conditions of a rolling schedule and a fixed lead time. These studies are acknowledged as the pioneering works of the fuzzy lot sizing problems. A significant number of studies have applied the fuzzy sets theory and techniques to develop and solve the production/inventory problems following these pioneering works. Some noteworthy instances include Park [42] and Vujosevic et al. [56] proposing a fuzzy EOQ model where unit ordering cost and unit holding cost are fuzzy. Chen et al. [12] extended the fuzzy EOQ model with backorder by fuzzifying the demand and the backorder cost. Roy and Maiti [46] also presented a fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. In the proposed EOQ model of Yao et al. [63], both order quantity and total demand were fuzzified using triangular fuzzy numbers. Buckley et al. [8] extended the problem into a multi-period/single item case, and solved the problem by implementing an evolutionary algorithm under three scenarios i) the demand is known for each period ii) the demand is unknown and fuzzy for each period, and iii) the demand is fuzzy and backordering is allowed. Above mentioned studies agree that fuzzy models reflect the real world conditions more rigorously. These studies also ascertain that, although the defuzzified total cost is higher compared to the crisp model, the fuzzy model is more utile since it considers the actual disturbances in the production system [26,29].

This study examines the lot sizing rules for the operating environment where the production process can be kept warm at some cost if production quantity in a period exceeds a fuzzy threshold value. We believe that our contributions lie in adapting fuzzy lot