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Similarity detection of rational space curves

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ABSTRACT

We provide an algorithm to check whether two rational space curves are related by a similarity. The algorithm exploits the relationship between the curvatures and torsions of two similar curves, which is formulated in a computer algebra setting. Helical curves, where curvature and torsion are proportional, need to be distinguished as a special case. The algorithm is easy to implement, as it involves only standard computer algebra techniques, such as greatest common divisors and resultants, and Gröbner basis for the special case of helical curves. Details on the implementation and experimentation carried out using the computer algebra system Maple 18 are provided.

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1. Introduction

Two objects are *similar* when one of them is the result of applying an isometry and scaling to the other. Therefore, two similar objects have the same shape, although their position and size can be different. Because of this, recognizing similar objects is important in the field of Pattern Recognition, where one typically has a database of objects and wants to compare, up to a similarity, a given object with all the elements in the database.

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Three-dimensional similarity detection is also important in Computer Graphics and Computer Vision, and therefore it has been addressed in a long list of papers. Following the introduction of Chen et al. (2003), the methods proposed in these papers can be grouped into two different categories: *shape-based* and *topology-based*. In the first category, one picks *feature descriptors* for the objects to be checked, giving rise to feature vectors that are later compared using appropriate metrics; see for instance the survey (Bustos et al., 2005) or the papers (Ankerst et al., 1999; Körtgen et al., 2003; Osada et al., 2001). In the second category, which has gained attention in recent years, a "skeleton" is computed from each object, which is later used for comparison purposes; see Hilaga et al. (2001), Sundar et al. (2003). The aforementioned papers, and others that can be found in their bibliographies, focus on surfaces, upon which (almost) no structure is assumed. At most, some of these papers require the objects to be modeled by means of polyhedra, so that they are considered to be meshings of perhaps more complex shapes. Additionally, in these references similarity detection is usually considered only up to a certain tolerance, so that the criteria are approximate.

Our approach is different. First, we deal with exact one-dimensional objects with a strong structure, namely rational space curves defined by rational parametrizations. Furthermore, we exploit the structure of the space curves to check, in a deterministic fashion, whether they are similar, and to explicitly compute the similarities between both curves in the affirmative case. In order to do this, we build on previous work on similarities of plane curves (Alcázar et al., 2014a) and symmetries of plane and space curves (Alcázar et al., 2014b, 2015). As in these papers, we exploit the rationality of the curves to reduce the problem to the parameter space. Analogously to the algorithm in Alcázar et al. (2015), the algorithm in this paper is based on comparing curvatures and torsions. However, similarity has the additional substantial difficulty of determining the scaling. Interestingly, this forces us to distinguish as a special case the *helical curves*, i.e., space curves with proportional curvature and torsion.

The basic steps in the algorithm are as follows. If the two given rational space curves are similar, then there exists a rational function relating the parameter spaces conforming to the similarity between the ambient spaces of the curves. Under the hypothesis that the parametrizations of the curves are proper, i.e., injective for almost all points, this rational function is a Möbius transformation. In our algorithm one first computes candidates for the scaling constants and then candidates for the Möbius transformations. After this, the similarities between the curves can be computed. If the input curves are non-helical, then we have two independent conditions involving the curvatures and torsions of the curves, and from these conditions the scaling constant can be found. If the input curves are helical, then these two conditions are no longer independent, and a different approach based on a procedure in Alcázar et al. (2015) is provided.

As for plane curves (Alcázar et al., 2014a, §3.5) the method can be adapted to the case of piecewise rational space curves. Moreover, for a space of properly parametrized curves satisfying affine invariance and uniqueness of the control polygon, we show that detecting similarity of such curve segments reduces to detecting similarity of the control polygons. This includes Bézier curves and, under certain conditions, B-spline curves and NURBS curves.

The structure of the paper is as follows. In Section 2 we provide some background on isometries, similarities, differential invariants and helical curves, and we prove some results that are needed later in the paper. Section 3 describes the algorithm for solving the problem, separately considering the case of non-helical and helical curves. In Section 4 we report on the experimentation with the algorithm, implemented in the computer algebra system Maple 18. In Section 5 we briefly discuss similarity detection of curve segments. Finally, conclusions and future work are presented in Section 6.

2. Background

2.1. Similarities and isometries of Euclidean space

A *similarity* of Euclidean space is a linear affine map from the space to itself that preserves ratios of distances. Equivalently, a map $f:\mathbb{R}^3 \longrightarrow \mathbb{R}^3$ is a similarity if and only if

$$f(\mathbf{x}) = \lambda \boldsymbol{Q}\,\mathbf{x} + \boldsymbol{b}, \quad 0 \neq \lambda \in \mathbb{R},\ \boldsymbol{b} \in \mathbb{R}^3,\ \boldsymbol{Q} \in \mathbb{R}^{3\times 3},\ \boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q} = \boldsymbol{I},\ \det(\boldsymbol{Q}) = 1, \tag{1}$$

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