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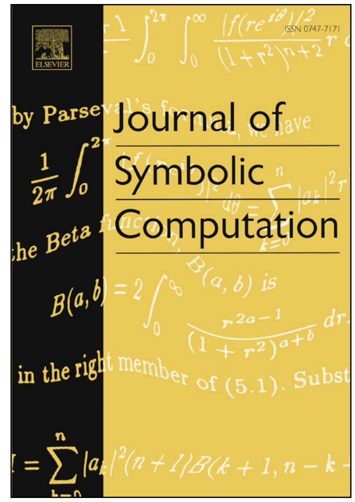

Solving generic nonarchimedean semidefinite programs using stochastic game algorithms

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Abstract

A general issue in computational optimization is to develop combinatorial algorithms for semidefinite programming. We address this issue when the base field is *nonarchimedean*. We provide a solution for a class of semidefinite feasibility problems given by generic matrices. Our approach is based on tropical geometry. It relies on tropical spectrahedra, which are defined as the images by the valuation of nonarchimedean spectrahedra. We establish a correspondence between generic tropical spectrahedra and zero-sum stochastic games with perfect information. The latter have been well studied in algorithmic game theory. This allows us to solve nonarchimedean semidefinite feasibility problems using algorithms for stochastic games. These algorithms are of a combinatorial nature and work for large instances.

Keywords: Semidefinite programming; nonarchimedean fields; tropical geometry; stochastic mean payoff games

1. Introduction

Semidefinite programming consists in optimizing a linear function over a spectrahedron. The latter is a subset of $\mathbb{R}^n$ defined by linear matrix inequalities, i.e., a set of the form

$$\mathcal{S} = \{x \in \mathbb{R}^n : Q^{(0)} + x_1 Q^{(1)} + \cdots + x_n Q^{(n)} \succcurlyeq 0\}$$

where the $Q^{(k)}$ are symmetric matrices of order m, and $\succcurlyeq$ denotes the Loewner order on the space of symmetric matrices. By definition, $X \succcurlyeq Y$ if and only if $X - Y$ is positive semidefinite.

Semidefinite programming is a fundamental tool in convex optimization. It is used to solve various applications from engineering sciences, and also to obtain approximate solutions or bounds for hard problems arising in combinatorial optimization and semialgebraic optimization. We refer the reader to Blekherman, Parrilo, and Thomas (2013) and Gärtner and Matoušek (2012) for information.

Semidefinite programs are usually solved via interior point methods. The latter provide an approximate solution in a polynomial number of iterations, provided that a strictly feasible initial solution, i.e., a point belonging to the interior of the set $\mathcal{S}$, is known. We refer the reader to de Klerk and Vallentin (2016) for a detailed analysis of the complexity in the bit model of interior point methods for semidefinite programming, and for a discussion of earlier complexity results based on the ellipsoid method.

Semidefinite programming becomes a much harder matter if one requires an exact solution. The feasibility problem (deciding the emptiness of the set $\mathcal{S}$) belongs to $\mathrm{NP}_{\mathbb{R}} \cap \mathrm{coNP}_{\mathbb{R}}$, where the subscript $\mathbb{R}$ refers to the BSS model of computation (Ramana, 1997). It is not known to be in NP in the bit model. A difficulty here is that all feasible points may have entries of absolute value doubly exponential in the size of the input. Also, there may be no rational solution (Scheiderer, 2016). Beyond their theoretical interest, exact algorithms for semidefinite programming may be useful to address problems of formal proofs, which sometimes lead to challenging (degenerate)

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