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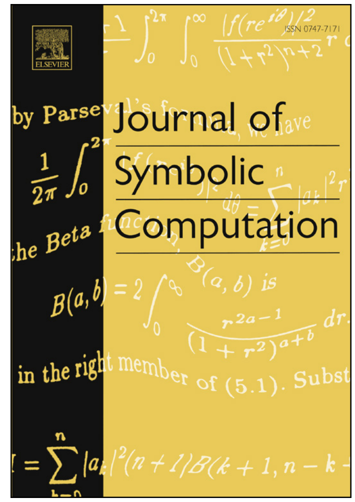

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Positive Root Isolation for Poly-Powers by Exclusion and Differentiation

Cheng-Chao Huang[a,b], Jing-Cao Li[a,b], Ming Xu[b,c], Zhi-Bin Li[b]

[a]Shanghai Key Lab of Multidimensional Information Processing, East China Normal University, Shanghai, China
[b]Department of Computer Science and Technology, East China Normal University, Shanghai, China
[c]Software Modeling and Verification Group, RWTH Aachen University, Aachen, Germany

Abstract

We consider a class of univariate real functions—poly-powers—that extend integer exponents to real algebraic exponents for polynomials. Our purpose is to isolate positive roots of such a function into disjoint intervals, each contains exactly one positive root and together contain all, which can be easily refined to any desired precision. To this end, we first classify poly-powers into simple and non-simple ones, depending on the number of linearly independent exponents. For the former, based on Gelfond–Schneider theorem, we present two complete isolation algorithms—exclusion and differentiation. For the latter, their completeness depends on Schanuel's conjecture. We implement the two methods and compare them in efficiency via a few examples. Finally the proposed methods are applied to the field of systems biology to show the practical usefulness.

Keywords: real root isolation; generalized polynomial; transcendental number; interval arithmetic; systems biology

1. Introduction

Solving nonlinear equations captures a central position in Computer Algebra with extremely wide applications to many fields, such as physics, economics, control theory and systems biology. The foundation of equation-solving is locating real roots of univariate functions. Since those roots are not rational (even not algebraic) in general, what one can really hope for is to isolate them into disjoint intervals, each contains exactly one real root and together contain all, which can be easily refined to any desired precision for further use.

Polynomials are the principal objects for real root isolation. The early work can date back to the 18th century. Isaac Newton proposed a numerical iteration method for solving $\mathbb{R}$-polynomial equations (Newton, 1711). The iterative values could approach real roots at quadratic convergence rate. But it depended on the choosing of starting points. Later a large number of modifications were proposed and applied to solve nonlinear equations and systems. With the hypothesis of differentiability, Smale (1986) improved Newton's method by deducing consequences from data at a single point. This point of view had valuable features for computation. Newton's

Email addresses: ecnucchuang@126.com (Cheng-Chao Huang), ecnujcli@126.com (Jing-Cao Li), mxu@cs.ecnu.edu.cn (Ming Xu), lizb@cs.ecnu.edu.cn (Zhi-Bin Li)