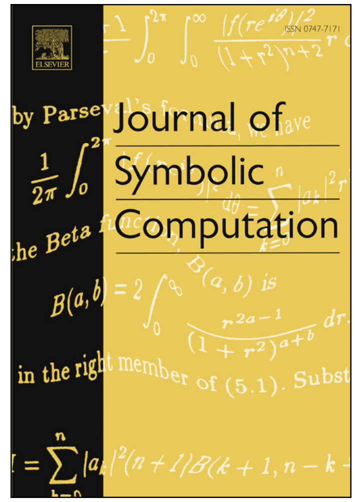

Algebraic diagonals and walks: Algorithms, bounds, complexity

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Algebraic Diagonals and Walks: Algorithms, Bounds, Complexity

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Abstract

The diagonal of a multivariate power series F is the univariate power series $\operatorname{Diag} F$ generated by the diagonal terms of F. Diagonals form an important class of power series; they occur frequently in number theory, theoretical physics and enumerative combinatorics. We study algorithmic questions related to diagonals in the case where F is the Taylor expansion of a bivariate rational function. It is classical that in this case $\operatorname{Diag} F$ is an algebraic function. We propose an algorithm that computes an annihilating polynomial for $\operatorname{Diag} F$. We give a precise bound on the size of this polynomial and show that, generically, this polynomial is the minimal polynomial and that its size reaches the bound. The algorithm runs in time quasi-linear in this bound, which grows exponentially with the degree of the input rational function. We then address the related problem of enumerating directed lattice walks. The insight given by our study leads to a new method for expanding the generating power series of bridges, excursions and meanders. We show that their first N terms can be computed in quasi-linear complexity in N, without first computing a very large polynomial equation.

1. Introduction

The *diagonal* of a multivariate power series with coefficients $a_{i_1,\dots,i_k}$ is the univariate power series with coefficients $a_{i,\dots,i}$. Particularly interesting is the class of diagonals of *rational* power series (i.e., Taylor expansions of rational functions). In particular, diagonals of *bivariate* rational power series are always roots of nonzero bivariate polynomials (i.e., they are algebraic series) [21, 35]. This property persists for multivariate rational power series, but only in positive characteristic, while the converse inclusion — algebraic series being diagonals of rational series — always holds [19, 21, 37]. As far as we are aware, the first occurrence of this result in the literature is an article of Pólya's [35], which deals with a particular class of bivariate rational functions; the proof uses elementary complex analysis. Along the lines of Pólya's approach, Furstenberg [21] gave a (sketchy) proof of the general result, over the field of complex numbers; the same argument has been enhanced later [25],[39, §6.3]. Three more different proofs exist: a purely algebraic

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