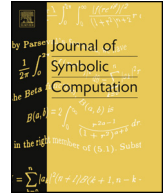




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A tropical construction of a family of real reducible curves

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ABSTRACT

We give a constructive proof using tropical modifications of the existence of a family of real algebraic plane curves with asymptotically maximal numbers of even ovals.

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1. Introduction

Let A be a non-singular real algebraic curve in $\mathbb{CP}^2$. Its real part, denoted by $\mathbb{R}A$, is a disjoint union of embedded circles in $\mathbb{RP}^2$. A component of $\mathbb{R}A$ is called an *oval* if it divides $\mathbb{RP}^2$ in two connected components. If the degree of the curve A is even, then the real part $\mathbb{R}A$ is a disjoint union of ovals. An oval of $\mathbb{R}A$ is called *even* (resp., *odd*) if it is contained inside an even (resp., odd) number of ovals. Denote by p (resp., n) the number of even (resp., odd) ovals of $\mathbb{R}A$.

Petrovski inequalities. For any real algebraic plane curves of degree $2k$, we have

$$-\frac{3}{2}k(k-1) \leq p-n \leq \frac{3}{2}k(k-1)+1.$$

One can deduce upper bounds for p and n from Petrovski inequalities and Harnack theorem (which gives an upper bound for the number of components of a real algebraic curves with respect to its genus):

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$$p \leq \frac{7}{4}k^2 - \frac{9}{4}k + \frac{3}{2},$$

and

$$n \leq \frac{7}{4}k^2 - \frac{9}{4}k + 1.$$

In 1906, V. Ragsdale formulated the following conjecture.

Conjecture (*Ragsdale*). For any real algebraic plane curve of degree $2k$, we have

$$p \leq \frac{3}{2}k(k-1) + 1,$$

and

$$n \leq \frac{3}{2}k(k-1).$$

In 1993, [Itenberg \(1995\)](#) used combinatorial patchworking to construct, for any $k \geq 5$, a real algebraic plane curve of degree $2k$ with

$$p = \frac{3}{2}k(k-1) + 1 + \left\lfloor \frac{(k-3)^2 + 4}{8} \right\rfloor,$$

and a real algebraic plane curve of degree $2k$ with

$$n = \frac{3}{2}k(k-1) + \left\lfloor \frac{(k-3)^2 + 4}{8} \right\rfloor.$$

This construction was improved by [Haas \(1995\)](#) then by [Itenberg \(2001\)](#) and finally by [Brugallé \(2006\)](#). Brugallé constructed a family of real algebraic plane curves with

$$\lim_{k \rightarrow +\infty} \frac{p}{k^2} = \frac{7}{4},$$

and a family of real algebraic plane curves with

$$\lim_{k \rightarrow +\infty} \frac{n}{k^2} = \frac{7}{4}.$$

Brugallé's construction split into two parts. First, he proved the existence of a family of real reducible curves $\mathcal{D}_n \cup \mathcal{C}_n$ in Σ_n , the n th Hirzebruch surface. The curve $\mathcal{D}_n$ has Newton polytope

$$\Delta_n = \text{Conv}((0, 0), (n, 0), (0, 1)),$$

the curve $\mathcal{C}_n$ has Newton polytope

$$\Theta_n = \text{Conv}((0, 0), (n, 0), (0, 2), (n, 1)),$$

and the chart of $\mathcal{D}_n \cup \mathcal{C}_n$ is homeomorphic to the one depicted in [Fig. 1](#). We refer to [Viro \(1984\)](#) for the definition of a chart.

Then, he used Viro's patchworking ([Viro, 1984](#)) to glue together many copies of the curve obtained by small perturbation of $\mathcal{D}_n \cup \mathcal{C}_n$. Viro's patchworking is a constructive method.

Brugallé's construction of the family $\mathcal{D}_n \cup \mathcal{C}_n$ used *real rational graphs theoretical method*, based on Riemann's existence theorem ([Brugallé, 2006](#); [Orevkov, 2003](#)). In particular, this method is not constructive. In this note, we give a constructive method to get such a family using tropical modifications and combinatorial patchworking for complete intersections ([Sturmfels, 1994](#)) (see also [Theorem 3](#)). In [Section 2](#), we recall basic definitions in tropical geometry. For more details about tropical geometry, we refer to [Mikhalkin \(2006\)](#). In [Section 3](#), we recall the notion of amoebas, the approximation of tropical hypersurfaces by amoebas and the notion of a real phase on a tropical hypersurface. In [Section 4](#), we recall the notion of a tropical modification along a rational function. In [Section 5](#) we give our strategy to construct the family $\mathcal{D}_n \cup \mathcal{C}_n$ and in [Sections 6 and 7](#), we explain the details of our construction.

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