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Perturbation results on the zero-locus of a polynomial

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A B S T R A C T

Let f and g be complex multivariate polynomials of the same degree. Extending Beauzamy's results which hold in the univariate case, we bound the Euclidean distance of points belonging to the zero-loci of f and g in terms of the Bombieri norm of the difference $g - f$. We also discuss real perturbations of real polynomials.

0. Introduction

In this work we address the problem of evaluating how much the zero-locus of a polynomial varies, if some perturbations on the polynomial coefficients are permitted. We start with a simple illustrative example where we explicitly show that, locally, it is possible to bound the Euclidean distance of points belonging to two different algebraic plane curves of equations $f = 0$ and $g = 0$ in terms of the difference between f and g measured using a suitable norm in the space of polynomials.

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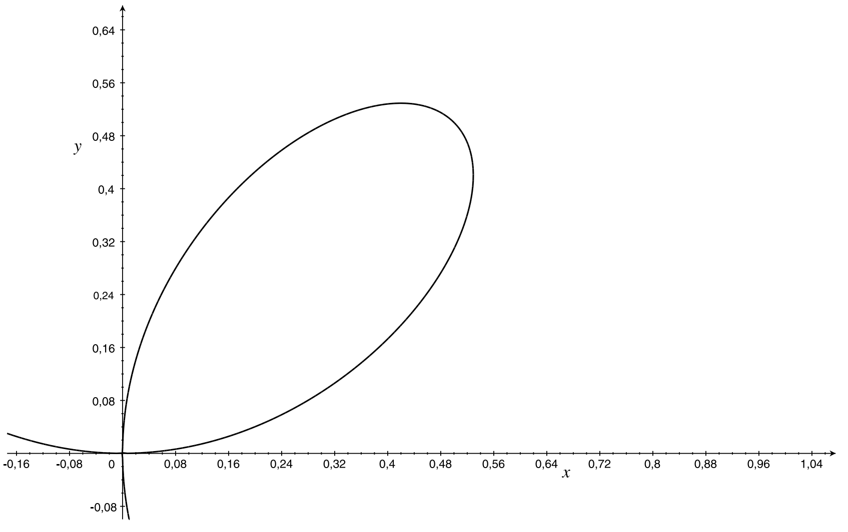

Fig. 1. The curve $f = 0$.

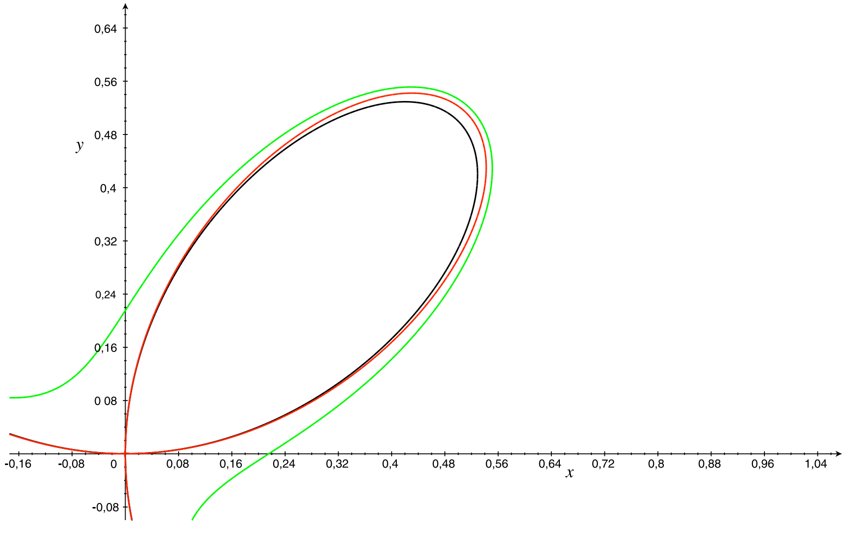

Fig. 2. The three curves $f = 0$ (black), $g_1 = 0$ (red), and $g_2 = 0$ (green). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Example 0.1 *(Descartes Folium).* In the affine plane $\mathbb{A}^2_{(x,y)}(\mathbb{R})$ consider the cubic curve $\mathcal{C}$ of equation $f(x, y) = 0$, where $f(x, y) = x^3 + y^3 - xy$ (see Fig. 1). Further, consider two other cubic curves $\mathcal{C}_1 : g_1(x, y) = 0$ and $\mathcal{C}_2 : g_2(x, y) = 0$, where $g_1(x, y) = x^3 + y^3 - (1 + \frac{\sqrt{6}}{100})xy$ and $g_2(x, y) = x^3 + y^3 - xy - \frac{1}{100}$ (see Fig. 2).

Note that the coefficients of the polynomials $g_1(x, y)$ and $g_2(x, y)$ "slightly" differ from the corresponding coefficients of the polynomial $f(x, y)$, and this leads to consider the curves $\mathcal{C}_1$ and $\mathcal{C}_2$ as "small" perturbations of the curve $\mathcal{C}$. In order to quantify the size of each perturbation, we need to have a measure of the difference between the "original" curve $\mathcal{C}$ and each "perturbed" curve $\mathcal{C}_1$ or $\mathcal{C}_2$. This is feasible, for instance, by computing the Bombieri norm (introduced in Definition 2.1) of the polynomials differences, that is, $g_1 - f$ or $g_2 - f$. In this case, the norm is always equal to 0.01,