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A projection-based compromising method for multiple criteria decision analysis with interval-valued intuitionistic fuzzy information

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ABSTRACT

The purpose of this paper is to develop a projection-based compromising method for addressing multiple criteria decision-making problems based on interval-valued intuitionistic fuzzy sets. The concept of projections considers not only the distance but also the included angle between evaluative ratings of alternative actions with respect to a criterion. In the interval-valued intuitionistic fuzzy context, this paper determines the respective projections of the evaluative ratings of each alternative on the positive-ideal and negative-ideal solutions and explores several essential properties. Next, this paper introduces the concepts of projection-based compromising indices and comprehensive compromising indices and further investigates relevant theorems for supporting the usefulness of these indices. Additionally, this paper proposes the projection-based comparative index and the comprehensive comparative index to serve as benchmark values for the comparison purpose. The improvement percentage of the comprehensive compromising value is acquired to determine the priority order of the alternatives, including the complete ranking order and the approval status for each alternative. The feasibility and the applicability of the proposed method are validated with an application problem of watershed site selection. Finally, several comparative analyses are conducted to verify the effectiveness and advantages of the proposed method over other relevant compromising decision-making methods.

1. Introduction

Multiple criteria decision-making problems address the ranking of alternatives and the selection of the best compromise alternative among a finite set of alternatives based on a finite set of criteria [18,37]. Decision makers are not always certain about their given decision or preference information and they often have some degree of uncertainty [29]. Moreover, exact data may be difficult to be precisely determined since human judgments are often vague under many conditions [15]. Thus, uncertain and imprecise assessment of information often occurs in practical decision situations [48,53], especially with respect to a lack of knowledge and experience, intangible and non-monetary criteria, the decision maker's limited information-processing capability, and a complex socio-economic environment [7,9,10,57]. Under these circumstances, dealing with imprecision or uncertainties in subjective judgments and evaluations becomes increasingly complex and important in multiple criteria decision analysis (MCDA) [10,27,51]. Accordingly, an extension to the fuzzy environment is a natural generalization of MCDA methods.

In fuzzy community, Atanassov and Gargov [5] introduced the concept of interval-valued intuitionistic fuzzy (IVIF) sets, which is an extension of intuitionistic fuzzy sets [1,2]. Intuitionistic fuzzy sets have better flexibility and practicality in the treatment of fuzzy information and uncertainty than ordinary fuzzy sets [6,23–25]. However, it is not completely justifiable or technically sound to quantify degrees of membership and non-membership in terms of a single numeric value in human cognitive and decision-making activities [25,26,34]. IVIF sets provide an effective and convenient way to resolve the difficulty. IVIF sets are characterized by the degree of membership, the degree of non-membership, and the degree of non-determinacy whose values are intervals rather than exact numbers [3,4]. Accordingly, IVIF sets are better than intuitionistic fuzzy sets for their great ability to handle imprecise and ambiguous information in practical applications [8,11,12,38]. Real-world decisions always require the use of more precise and accurate data [20]; thus, IVIF sets are appropriate

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for addressing MCDA problems within a complex and uncertain environment. IVIF sets have been applied productively in the MCDA field [7–9,11,12,14,16,17,19,28,30,31,36,39,52,55].

Compared with the existing MCDA methods based on IVIF sets, comparatively little research has focused on the development of the projection methods and techniques within the IVIF environment. The projection method [45–47,49] is a much better measure than the distance measure that has been widely used in many aspects of the MCDA methodology. The concept of projections considers not only the distance but also the module and included angle between objects evaluated [44,47,49]. Thus, the replacement for the distance measure by the projection measure can more fully consider the separation between two objectives measured. Accordingly, the existing MCDA methods and techniques would be evidently improved if the distance measure or other separation measures were replaced by the projection measure. Approaches that use projection techniques have been applied successfully to address multiple criteria decision-making problems [41,44–47,49,50,52,54,56]. However, little attention has been given to the employment of the projection methods for MCDA in the IVIF context. For example, the projections used in Yue [45] and Zheng et al. [56] were defined on ordinary numbers. The projections used in Wang et al. [41], Yue [46], and Yue and Jia [50] were defined on interval numbers. Consider the projection methods in intuitionistic fuzzy and/or IVIF contexts. Xu and Hu [44] established two projection models to measure the similarity degrees between each alternative and the intuitionistic fuzzy ideal point, and between each alternative and the IVIF ideal point for ranking alternatives. In a manner similar to the technique for order preference by similarity to ideal solution (TOPSIS), Yue [47] established a relative closeness of each alternative decision with respect to their projections on ideal decisions to solve the partner selection problem with linguistic values and intuitionistic fuzzy information. Based on intuitionistic trapezoidal fuzzy numbers, Zhang et al. [54] used the information entropy method to determine the criterion weight and proposed the grey relational projection method to rank alternatives. Zeng et al. [52] proposed a projection method to derive experts' weights and handle multiple criteria group decision-making problems with intuitionistic fuzzy sets and IVIF sets. Yue and Jia [49] developed an extended TOPSIS method for group decision making with hybrid intuitionistic fuzzy information. In these works, the projections used in Xu and Hu [44], Yue [47], Zeng et al. [52], and Zhang et al. [54] were defined on intuitionistic fuzzy numbers. In particular, Xu and Hu [44], Yue and Jia [49], and Zeng et al. [52] also investigated the projections defined on IVIF numbers. On the whole, within the literature on the decision-making methods in the IVIF context, notably few attempts have been made to develop a projection comparison approach for solving MCDA problems with IVIF sets. The theory of IVIF sets is useful and valuable for depicting uncertainty and managing imprecision in decision information. From this perspective, this paper attempts to propose a new compromising decision-making method based on projections between IVIF numbers for addressing MCDA problems within the decision environment of IVIF sets.

The purpose of this paper is to develop a projection-based compromising method for MCDA with IVIF information. First, this paper explores several essential properties of the cosine of the included angle between the evaluative ratings of an alternative and the ideal solution. Additionally, the respective projections of the evaluative ratings of an alternative on the positive-ideal and negative-ideal solutions are proposed, and some important properties are investigated as well. Next, this paper introduces new useful concepts of projection-based compromising indices and comprehensive compromising indices. Moreover, some relevant theorems are developed to support these indices. This paper employs a newly-developed measure proposed by Guo [17] to define the comprehensive compromising value for each alternative. To compare the comprehensive compromising value with the benchmark value, this paper further proposes the concepts of projection-based comparative indices and comprehensive comparative indices. Considering the comprehensive comparative value as a benchmark value for the comparison purpose, this paper establishes the improvement percentage of the comprehensive compromising value for each alternative. All of the alternatives can be ranked according to the descending order of the improvement percentages. Furthermore, the alternatives can be differentiated into the better compromise choices that are approved and preferred to recommend and the worse compromise choices that are not approved to recommend. To demonstrate the feasibility and the applicability of the developed methods, an illustrative application is employed to explore the problem of watershed site selection. This paper also conducts some comparisons with other relevant compromising decision-making methods and validates the effective use of the proposed method.

The remainder of this paper is organized as follows. Section 2 briefly reviews the concepts of IVIF sets and introduces the concept of projections. Section 3 describes an MCDA problem in an IVIF setting and develops a new compromising decision-making method using a projection-based compromising approach for addressing MCDA problems involving IVIF information. Section 4 demonstrates the feasibility and the applicability of the proposed methodology using a practical application, i.e., the selection of watershed sites. This section also compares the solution results yielded by other relevant compromising methods with those of the proposed method. Finally, Section 5 presents the conclusions.

2. Preliminaries

Definition 1. [3–5] Let X be a finite universal set. Let Int([0,1]) denotes the set of all closed subintervals of the unit interval [0,1]. An IVIF set $\tilde{A}$ in X is an object that has the following form:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle | x \in X\}, \tag{1}$$

where the functions

$$\mu_{\tilde{A}} : X \to \text{Int}([0, 1]), x \in X \to \mu_{\tilde{A}}(x)(= [\mu^-_{\tilde{A}}(x), \mu^+_{\tilde{A}}(x)]) \subseteq [0, 1] \text{and} \tag{2}$$

$$\nu_{\tilde{A}} : X \to \text{Int}([0, 1]), x \in X \to \nu_{\tilde{A}}(x)(= [\nu^-_{\tilde{A}}(x), \nu^+_{\tilde{A}}(x)]) \subseteq [0, 1] \tag{3}$$

define the intervals of the degree of membership and the degree of non-membership of the element $x \in X$ to the set $\tilde{A}$, respectively, and satisfy the condition:

$$0 \leq \mu^+_{\tilde{A}}(x) + \nu^+_{\tilde{A}}(x) \leq 1 \quad \forall x \in X. \tag{4}$$

Definition 2. [3–5] For each IVIF set $\tilde{A}$ in X, the hesitation interval (or intuitionistic fuzzy interval) relative to $\tilde{A}$ for each $x \in X$ is defined as follows: