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Revision with probability ☆

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ABSTRACT

This paper investigates belief revision with uncertainty. Normally, this kind of revision is processed in the framework of possibilistic logic, which is good at processing incomplete and imprecise information. However, the possibilistic logic based revision does not provide a probabilistic explanation for uncertainty. To solve this problem, it is necessary to express this problem from a perspective of probability. This paper proposes the definitions of formula probability and model probability, and then derives the conversion equation between them. To deal with uncertain belief revision, a probabilistic estimation is used. The approaches for revision with a reliable input and an uncertain input are discussed respectively. At last, differences between our approach and the existing ones for uncertain revision are explored.

1. Introduction

Belief revision is essential to artificial intelligence and database, which describes the process of information changing when new introduced information contradicts with the existing information. In 1985, Alchourrón, Gärdenfors and Makinson provide the basic postulates for revision operators in belief set (Alchourrón et al., 1985). Katsuno and Mendelzon reformulate these postulates and proposed KM theory to describe the properties that a revision operator for finite knowledge base should satisfy

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(Katsuno and Mendelzon, 1989). Besides the above rational analysis, different approaches for revising include: Dalal proposed a model-based method for belief set (Dalal, 1988), Nebel provided a set of syntax-based revision methods for finite sets (Nebel, 1991), and Wei Li proposed a logical framework to deal with revision problem by inference (Li, 1994). In all of these approaches, the correctness of formulas is deterministic, i.e. a formula is either true or false.

However, revision problems are not always deterministic (Spohn, 1988). Uncertain information pervades economics, engineering, and health/environmental sciences studies (Alola et al., 2013). As information science and internet technology develop, it appears in large amount on internet (Ma, 2011). Then how to obtain reliable information from uncertain one becomes important in information science. Probability logic, which takes uncertainty as probability, was first introduced to deal with uncertain information processing in 1986 (Nilsson, 1986). Revision approaches based on probability are provided (Nilsson, 1986; Schlecta, 1991). In 1988, Spohn proposes an ordinal condition function (OCF for short) based framework for deterministic revision (Spohn, 1988), and holds that this framework also has some properties of uncertain revision. Dubois et al. apply possibilistic logic to dealing with uncertain revision (Dubois and Prade, 1997; Benferhat et al., 2002; Qi, 2008). Possibilistic logic albeit good at processing incomplete or imprecise information, is still a reasoning tool based on ordering (Dubois et al., 1994). Therefore, the possibilistic logic based revision is essentially based on priorities (Benferhat et al., 2002; Benferhat et al., 2008), which do not use probabilistic property in information processing.

This paper views the problem of uncertain revision from a different perspective, and uncertainty by probability. Each formula is associated with a real number between $[0, 1]$. Different from the revision approaches in possibilistic logic, the number does not express the priority of a formula, but describes the probability of the formula being true. In our approach, the key is to estimate the probabilities of all possible models, among which the highest is used for formula selecting. Formulas that are satisfied by a selected model constitute a revision result. To accomplish this revision process, it is necessary to explain the probabilities of formula and model respectively, and derive the conversion equation between formula probabilities and model probabilities. With the assumption of independence, a probability estimation method is proposed according to probability theory. Our approach is different from the possibilistic logic based approach and the conventional probability based approach in many aspects, and a comparison between them will be displayed below.

This paper is organized as follows: Following a brief review of related works, Section 3 proposes the formal description of uncertain revision with probability. After that, we derive the conversion equation between formula probabilities and model probabilities in Section 4, and present the revision methods with reliable input and uncertain input respectively in Section 5. A comparison of our work with the existing approaches for uncertain revision is presented in Section 6. And the last section concludes this paper.

2. Related work

Mainly, there are two kinds of frameworks to solve revision problem with uncertainty (Dubois and Prade, 1993; Dubois and Prade, 1994). One is the framework using possibilistic logic, which characterizes the uncertainty by priority. This framework has an advantage in processing incomplete information or imprecise probabilities. While the other framework, which is based on probability and defines the uncertainty as probability, normally has a wider application to uncertain problems, as a lot of uncertain information is characterized by frequency in information technology, engineering, economics and also in real life problem (Alola et al., 2013).

The possibilistic logic based revision framework is widely studied in the last few years (Dubois and Prade, 1997; Benferhat et al., 2002; Benferhat et al., 2008; Qi, 2008). It provides each model/formula with a possibility, and revises a given possibility formula set according to the possibilities of formulas. But similar to the revision approach based on OCF or weighted formulas (Spohn, 1988; Dubois and Prade, 1997; Williams, 1994), it is still a reasoning tool based on ordering, with only differences of how the order is defined to tell them apart (Benferhat et al., 2002).

Let Ω be the set of all models. In the framework of possibilistic logic, a possibility distribution $\pi : \Omega \to [0, 1]$ is provided to show the compatibility degree of a model ω and the existing formulas.

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