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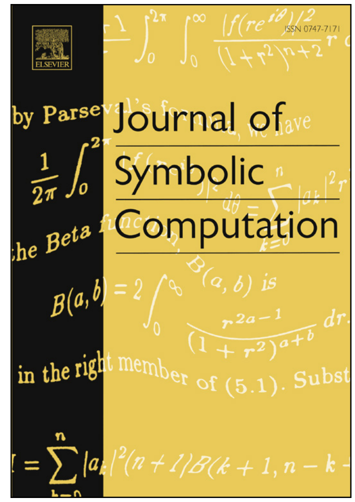

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Parallel degree computation for binomial systems

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Abstract

Solution sets of systems of binomial equations is of great interest in applied mathematics. For both theoretic and applied purposes, the degree of a solution set (its maximum number of isolated intersections with an affine space of complementary dimension) often plays an important role in understanding its geometric structure. This paper proposes a specialized parallel algorithm for computing the degree on GPUs that takes advantage of the massively parallel nature of GPU devices. The preliminary implementation shows remarkable efficiency and scalability when compared to the closest CPU-based counterpart. As a case study, the algorithm is applied to the master space problem of $\mathcal{N} = 1$ gauge theories. The GPU-based implementation achieves nearly 30 fold speedup over its CPU-only counterpart enabling the discovery of previously unknown results.

Keywords: Binomial Systems, Homotopy Continuation, Algebraic Geometry, BKK Root-count, GPU Computing, Supersymmetric gauge theories.

1. Introduction

The problem of solving a system of polynomial equations is an important problem in computational mathematics. Of special interest is the problem of solving systems of binomial equations, or simply binomial systems for they appear naturally in many applications and specialized algorithms exist (e.g., [27]). In many applications, only the solutions of a binomial system for which no variable is zero are needed. That is, we are only interested in solutions inside $(\mathbb{C}^*)^n$ where $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$. The collection of such solutions are known as the $\mathbb{C}^*$-solution set and will be the focus of this article. Such a $\mathbb{C}^*$-solution set of

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