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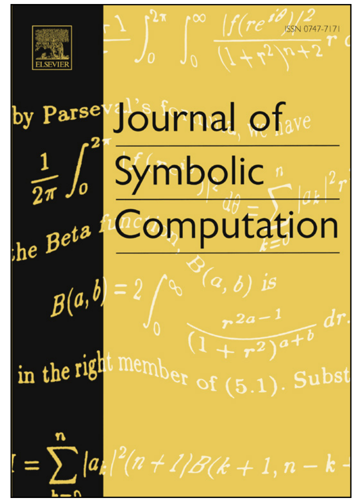

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Construction algorithms for rational cubic surfaces

Jon González-Sánchez*† and Irene Polo-Blanco‡§

Abstract

By a criterion of Swinnerton–Dyer it is known that a smooth cubic surface S defined over $\mathbb{Q}$ is birationally trivial over $\mathbb{Q}$ if and only if $S(\mathbb{Q}) \neq \emptyset$ and S contains a $\mathrm{Gal}(\overline{\mathbb{Q}}, \mathbb{Q})$-stable set of 2, 3 or 6 skew lines. In this text we describe birationally trivial smooth cubic surfaces over $\mathbb{Q}$ and provide algorithms to construct explicit cubic surfaces for each of the cases in Swinnerton–Dyer's criterion.

Introduction

Recent studies focus on the implementation of algorithms to parametrize algebraic surfaces ([5], [6], [7] and [8]). Some of them provide explicit algorithms for the parametrization of real cubic surfaces (compare [19], [1] and [16]). We recall that a cubic surface S is the vanishing set of a homogeneous polynomial f of degree 3 in $\mathbb{P}^3$, i.e.,

$$S = \{(x : y : z : t) \in \mathbb{P}^3 \mid f(x : y : z : w) = 0\}.$$

By a classical result of Clebsch [4] we know that a smooth cubic surface over the complex numbers is birationally trivial over $\mathbb{C}$. Even more, it admits a parametrization by cubic polynomials over $\mathbb{C}$ (see[4] or [9, Corollay 4.7 and Remarks 4.7.2]). If S is a smooth cubic surface defined over the real numbers, then S is birationally trivial over $\mathbb{R}$ if and only if the real locus of S is connected or, equivalently, if and only if S contains two disjoint real lines or two disjoint complex conjugate lines (compare [20], [11], [10] and [15]).

Concerning smooth cubic surfaces defined over the field of the rational numbers a criterion by Swinnerton–Dyer states that a smooth cubic surface S over $\mathbb{Q}$ is birationally trivial over $\mathbb{Q}$ if and only if the surface contains a $\mathrm{Gal}(\overline{\mathbb{Q}}, \mathbb{Q})$–stable set of 2, 3 or 6 skew lines and S has points defined over $\mathbb{Q}$ (see [21]).

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