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Reference line-based Estimation of Distribution Algorithm for many-objective optimization[☆]

Yanan Sun[a], Gary G. Yen[b,*], Zhang Yi[a]

[a] College of Computer Science, Sichuan University, Chengdu, Sichuan 610065, China
[b] School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, OK 74078, USA

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ABSTRACT

Multi-Objective Evolutionary Algorithms (MOEAs) are preferred in solving multi-objective optimization problems due to their considerable performance giving decision-maker a set of not only convergent but diversified promising solutions. However, the scalability of MOEAs deteriorates in addressing many-objective optimization problems which involve more than three conflicting objectives. The principal reason is largely due to the deficiency of the existing genetic operators which cannot generate promising offspring from parents chosen by the Pareto-dominance rule in these MOEAs. Estimation of Distribution Algorithms (EDAs) generate offspring with a probabilistic model built from the statistics extracting upon existing solutions to expectedly alleviate the weakness arisen in genetic operators. In this paper, a reference line-based EDA is proposed for effectively solving many-objective optimization problems. Specifically, the estimation model is built based on the reference lines in the decision space to sample solutions with *favorable proximity*. Then solutions with *considerable diversity* in Pareto-optimal front are selected. These two phases collectively promote the needed convergence and diversity for the proposed algorithm. To evaluate the performance, extensive experiments are performed against four state-of-the-art many-objective evolutionary algorithms and two EDAs over DTLZ and WFG test suites with 5-, 8-, 10-, and 15-objective. Experimental results quantified by the selected performance metrics indicate that the proposed algorithm shows significant competitiveness in tackling many-objective optimization problems.

1. Introduction

A reference line-based Estimation of Distribution Algorithm (EDA) Many-Objective Evolutionary Algorithm (MaOEA) is proposed in this paper for efficiently tackling Many-objective Optimization Problems (MaOPs). Specifically, MaOEAs aim at solving m conflicting objectives concurrently and m is greater than three [1,2]. Generally, a given MaOP is mathematically formulated by Eq. (1):

$$f = (f_1(x), \dots, f_m(x))^T \qquad \text{subject to} \quad x \in \Omega \tag{1}$$

where $\Omega \in \Pi_{i=1}^{n}\left[x_i^l, x_i^u\right]$ is the feasible space and $-\infty < x_i^l \leq x_i^u < \infty$ for $i = 1, \dots, n$. The mapping f from Ω to $\mathbb{R}^m$ defines m conflicting objective functions $f_i(x)$ and $i = 1, \dots, m$. Without loss of generality, it is assumed that $f_1(x), \dots, f_m(x)$ are to be minimized, as maximization problems can be transformed into minimization problems by duality principle. Because MaOPs widely exist in many real-world applications, such as calibration problems of automotive engine with 10-objective [3], management in land exploitation with 14-objective [4], to name a few, there is a strong incentive for efficiently and effectively solving MaOPs.

In contrast to a single-objective optimization problem in which one single global optimal solution is desired, a set of trade-off solutions, namely Pareto-optimal solutions, is preferred in a MaOP to give decision-makers more alternatives for their preferences due to the conflicting nature among the objectives. Moreover, the Pareto-optimal solutions in the decision space constitute the Pareto-optimal Set (PS) while their images correspond to the Pareto-optimal Front (PF) in the objective space [5]. MaOEA has been recognized as one of the promising paradigms to address MaOPs due to its population-based nature and meta-heuristics search ability to be capable of obtaining a set of solutions approximating the PF in a single run. Generally speaking, MOEAs aim at pursuing two distinct, yet complement, goals throughout the evolution process:

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* Corresponding author.
E-mail addresses: yanan_sun@outlook.com (Y. Sun), gyen@okstate.edu (G.G. Yen), zhangyi@scu.edu.cn (Z. Yi).

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1) improving the convergence and 2) preserving the diversity among solutions.

1.1. Introductions to algorithms in solving MaOPs

During the past several decades, numerous Multi-Objective Evolutionary Algorithms (MOEAs), such as elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) [6], advanced version of Strength Pareto Evolutionary Algorithm (SPEA2) [7], have been developed for dealing with Multi-objective Optimization Problems (MOPs) [8,9] in which two or three objectives are to be optimized simultaneously [1,2]. However, these MOEAs severely degrade their performance in handing MaOPs. The major reason is the loss of selection pressure, i.e., a large proportion of individuals is all non-dominated, which leads to the Pareto-based primary selection approach losing its selecting effectiveness [10]. Furthermore, most of these solutions are only the bests at very few objectives but fairly worse in others, which is known as the Dominance Resistant (DR) solutions [11–13]. As a result, the density-based secondary diversity improving mechanism is activated, which is recognized as the Active Diversity Promotion (ADP) [12], to determine which solutions survive into the next generation. Due to the DR and ADP phenomena, the selected solutions are not moving uniformly towards the PF as the evolution continues and could even be stagnated at far away [14].

To this end, various evolutionary algorithms for solving MaOPs, namely MaOEAs,[1] have been specifically designed for addressing MaOPs. In summary, these MaOEAs manage the adverse impact of DR and ADP from different schemes: 1) exploiting novel approaches to strengthen the diversity of selected solutions, and 2) enhancing the comparison mechanism of the traditional dominance relationship. For example, the reference point-based Non-dominated Sorting Genetic Algorithm for many-objective optimization (NSGA-III) [15] improves the diversity by checking which one has the nearest perpendicular distance to the reference lines, and these reference lines are manually set or evenly sampled in the unit hyper-plane of the objective space. The Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [16] decomposes one considered MaOP into a set of scalar optimization subproblems, and then optimizes them simultaneously by a group of predefined well distributed weight vectors aggregating all objectives. Especially, the diversity of MOEA/D is maintained by these weight vectors which direct the population towards different areas of the PF. Furthermore, the dominance comparison occurs only in the neighboring solutions, which reduces the adverse impact of the DR to some extent. The Grid-based Evolutionary Algorithm for many-objective optimization (GrEA) [17] employs the grid-based fitness comparison technique to relax the dominance relationship of solutions. Other grid-based quantitative measurements are also incorporated into the fitness value to strengthen the diversity in mating and environmental selections. In the Hypervolume-based many-objective Evolutionary algorithm (HypE) [18], the selection is preferred given the fitness assigned by the corresponding hypervolume contribution which is estimated by the Monte Carlo simulation. This fitness assignment concurrently avoids the deficiency of traditional dominance comparison mechanism and improves the diversity. In addition, the same selection principle is also employed in its mating selection.

1.2. Launching of EDAs

In evolutionary algorithms, individuals evolving towards the regions of the promising solutions are generated from their elite parents by genetic operators, such as crossover and mutation. Recently, existing genetic operators have been extensively investigated over a set of scalable benchmark test problems, which concludes that existing genetic operators are crucial to the performance of MOEAs, and cannot guarantee promising solutions to be generated especially in MaOPs [19]. Furthermore, most genetic operators require a set of parameters to be empirically assigned in advance, such as the probabilities of crossover and mutation, the distribution indexes in widely used simulated binary crossover (SBX) [20] and polynomial mutation [21] operators. As a consequence, these deficiencies of the existing genetic operators collectively motive the development of the Estimation of Distribution Algorithm (EDA) [22–25] which is a kind of computing paradigm to solve MOPs [26–29] by generating new promising offspring with the probabilistic models built based on the solutions the algorithms have visited.

Typically, EDAs-based MOEAs are classified into two distinct categories based on their employment of estimation models. The first category is often known as the mixture probability model-based EDAs. For instance, the multi-objective mixture-based iterated density estimation evolutionary algorithm [26] utilized the mixed probability distributions to sample well-distributed solutions and the multi-objective Parzen-based EDA [30] learnt from the Gaussian and Cauchy kernels to build its models. In [31], the multi-objective hierarchical Bayesian optimization algorithm was designed by the mixture Bayesian network-based probabilistic model for discrete multi-objective optimization problems. In addition, the multi-objective extended compact genetic algorithm [32] employed a marginal product model as the mixture of probability model. Furthermore, a Regularity-based Model EDA (RM-MEDA) was proposed lately in [28] in which the model is built based on the mixture normal distribution over the regularity. The other category covers the Bayesian network-based EDAs. Example include the multi-objective Bayesian optimization algorithm [33] utilizing the Bayesian Optimization Algorithm (BOA) to build a Bayesian network as its model for generating offspring. In addition, a related work was investigated in [34] to predict the model by strengthening Pareto ranking approach [35] and BOA. Furthermore, Laumanns and Ocenasek in [36] proposed a Bayesian multi-objective optimization algorithm whose model was built over the solutions selected by the ϵ-Pareto ranking method [37]. Moreover, an improved non-dominated sorting approach was employed by decision tree-based multi-objective EDA [38] to select a subset of solutions serving for a regression decision tree to learn the model. Recently, the Multi-dimensional Bayesian Network EDA (MBN-EDA) was proposed in [29] specifically for addressing MaOPs.

1.3. Motivation of the proposed algorithm

It is believed that EDAs are capable of solving MaOPs without suffering the weaknesses of MOEAs employing genetic operators. Although, MBN-EDAs have been tested to be with the ability in addressing MaOPs, the development of Many-objective Optimization EDAs (MaOEDA) is still in its infancy. Especially, probability models based on regularity have been extensively investigated in the discipline of statistical learning [39,40], and regularity-based models are easier to build, and fairly effective. In addition, regularity-based EDAs can visually help decision-makers to easily select their desirable solutions. Therefore, a variety of regularity-based MOEAs have recently been successfully proposed for dealing with MOPs [28,41,42]. Moreover, because uniformly reference points are capable of directing the search towards the exceptional solution locations, instead of a heuristic exploration in a large space of MaOPs, the reference points related paradigms have been successfully employed in MaOEAs, such as MOEA/D and NSGA-

[1] In this context, MaOEAs include the algorithms which are originally designed for MOPs, and now are extended for MaOPs.