Knowledge-Based Systems 000 (2017) 1–15

Contents lists available at ScienceDirect

Knowledge-Based Systems

journal homepage: www.elsevier.com/locate/knosys

A new approach for group decision making method with hesitant fuzzy preference relations

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ARTICLE INFO

Article history:
Received 18 July 2016
Revised 25 November 2016
Accepted 13 March 2017
Available online xxx

Keywords:
Group decision making
Hesitant fuzzy preference relation
Multiplicative consistency
Consensus
Distance measure

ABSTRACT

This paper focuses on group decision making with hesitant fuzzy preference relations (HFPRs). To derive the consistent ranking order, a new multiplicative consistency concept for HFPRs is introduced that considers all information offered by the decision makers. The main feature is that this concept neither adds values to hesitant fuzzy elements nor disregards any information provided by the decision makers. To judge the multiplicative consistency of HFPRs, 0–1 mixed programming models are constructed. According to the assumption that there is an independent uniform distribution on values in hesitant fuzzy elements, the hesitant fuzzy priority weight vector is derived from multiplicative consistent reciprocal preference relations and their probabilities. Meanwhile, several consistency based 0–1 mixed models to estimate missing values in incomplete HFPRs are constructed that can address the situation where ignored objects exist. Considering the consensus in group decision making, a distance measure based consensus index is defined, and a method for improving the group consensus is provided to address the situation where the consensus requirement is unsatisfied. Then, a distance measure between any two HFPRs is introduced that is used to define the weights of the decision makers. Furthermore, a multiplicative consistency and consensus based interactive algorithm for group decision making with HFPRs is developed. Finally, a multi-criteria group decision making problem with HFPRs is offered to show the concrete application of the procedure, and comparison analysis is also made.

1. Introduction

With the constant complexity of socioeconomic decision-making problems, preference relations with precise judgments [34,37] are suffering more and more restrictions. To extend the application of preference relations, researchers introduced fuzzy sets given by Zadeh [55] to preference relations and proposed several types of fuzzy preference relations, such as interval fuzzy preference relations [38,50], triangular fuzzy preference relations [42,51], trapezoidal fuzzy preference relations [8] and linguistic fuzzy preference relations [21,52]. However, all these types of fuzzy preference relations only give the decision makers (DMs)' preferred judgments. To both denote the preferred and non-preferred opinions on objects, Szmidt and Kacprzyk [39] introduced the concept of intuitionistic fuzzy preference relations, where the decision makers' judgments are denoted by using intuitionistic fuzzy values [6]. Later, Xu [54] further provided interval-valued intuitionistic fuzzy preference relations that permit the DMs to apply intervals rather than real numbers to express their preferred and non-preferred judgments. Taking the advantages of linguistic varibles and intuitionistic fuzzy values, Meng et al. [32] presented intuitionistic linguistic fuzzy preference relations (ILFPRs) and offered an additive consistency and consensus analysis based group decision making with ILFPRs.

Recently, Torra [41] noted that there might be several values for a pair of compared objects and provided hesitant fuzzy sets (HFSs). To denote the hesitancy of the DMs, Xia and Xu [48] introduced the concept of hesitant fuzzy elements (HFEs), by which the authors provided hesitant fuzzy preference relations (HFPRs) that can be seen as an extension of reciprocal preference relations (RPRs) [34]. According to Tanino's additive consistency concept, Zhu and Xu [64] applied the defined operations on HFEs to derive reduced RPRs that have the highest consistency level with respect to HFPRs. Considering weak consistency, the authors provided hesitant preference relations (HPRs) and hesitant reachability matrix (HRM), by which an algorithm is developed to derive the reduced RPRs. Using Tanino's multiplicative consistency concept, Zhu et al. [65] proposed two procedures: one procedure builds programming models to derive the crisp priority weight vector (called the α-

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http://dx.doi.org/10.1016/j.knosys.2017.03.010
0950-7051/© 2017 Published by Elsevier B.V.

normalization based method), which is derived from one of RPRs in HFPRs; the other extends all HFEs to the length of the one with the most number of possible values (called the β-normalization based method). Then, the authors defined the consistent HFPRs by considering the consistency of the ordered RPRs. According to the defined distance measure, the authors studied the threshold of consistency. Finally, the authors used the aggregation operators in [48] to calculate the ranking HFEs and applied the score function [48] to derive the ranking orders of objects. As Meng et al. [29,31] noted, different HFEs are derived when we add values to HFEs. Thus, the Hamming distance in [48] needs to be further studied. Zhu et al. [65] did not explain why we should use the ordered RPRs obtained from HFPRs to judge the consistency. Just as the authors said, the α-normalization based method removes elements from HFEs, and the β-normalization based method adds values to HFEs. These two procedures both derive different HFPRs because the elements in HFEs are changed. This means that the priority weight vector is not obtained from the original HFPRs. Zhang et al. [56,57] adopted Tanino's additive consistency concept and β-normalization based method to develop two methods for decision making with HFPRs. Xu et al. [49] developed two programming model based methods to calculate the crisp priority weight vector from incomplete HFPRs that are based on Tanino's additive and multiplicative consistency concepts. Although these models are based on the consistency analysis, they fail to address inconsistent case. Considering the incomplete case, following the work of Zhu et al. [65], Zhang [61] introduced two methods to group decision making with incomplete HFPRs: a α-normalization based method and a β-normalization based method. Note that the β-normalization based method requires all HFEs offered by the DMs to have the same length.

From the above analysis, one can find that all previous researches about HFPRs are based on RPRs and use Tanino's additive and multiplicative consistency concepts. The α-normalization based approaches [49,61,65] derive the crisp priority weight vector that cannot reflect the hesitancy of the DMs at all. Although the β-normalization based approaches [56,57,61,65] can obtain the hesitant fuzzy priority weight vector, they need add values to HFEs that derives different HFPRs with respect to the original ones. Furthermore, it is unsuitable to judge the consistency of HFPRs by only considering the ordered RPRs. To address these limitations, this paper defines a new multiplicative concept for HFPRs that uses Tanino's multiplicative consistency concept. In contrast to the previous consistency concepts, the new concept need not add or remove values provided by the DMs. To judge the consistency of HFPRs, 0–1 mixed programming models are established. Based on the assumption of uniform distribution, the probability of each RPR is determined. Then, we can derive the hesitant fuzzy priority weight vector from multiplicative consistent RPRs. After that, 0–1 mixed programming models to determine the missing values in incomplete HFPRs are constructed. Considering group decision making with HFPRs, a consistency and consensus based interactive algorithm is performed. To do this, the rest is organized as follows:

Section 2 reviews several related basic concepts, such as HFSs, HFPRs and the operational laws on HFEs. Then, it recalls several previous methods to derive the priority weight vector from HFPRs. Section 3 defines a new consistency concept and compares with several previous ones. Then, 0–1 mixed programming models are constructed to judge the consistency of HFPRs. Meanwhile, an algorithm to derive the hesitant fuzzy priority weight vector from HFPRs is provided. Subsequently, 0–1 mixed programing models to estimate missing values in incomplete HFPRs are established, which have the highest consistency level with respect to the known judgments. Section 4 offers a group consensus index using the defined distance measure. Then, a method to improve the group consensus is introduced. Based on the consistency and consensus analysis, an interactive algorithm to group decision making with HFPRs is developed. Section 5 applies a practical example to show the concrete application of the new procedure. Meanwhile, comparison analysis with several previous methods is performed. Section 6 lists the final conclusions and future remarks.

2. Basic concepts

Different from multiplicative preference relations, Orlovsky [34] proposed the concept of RPRs to give the relationship between each pair of objects.

Definition 1. [34]. A RPR R on a set of objects $X = \{x_1, x_2, \ldots, x_n\}$ is defined by $R = (r_{ij})_{n\times n}$ such that $r_{ij}+r_{ji} = 1$ with $r_{ij} \in [0, 1]$, where $i, j = 1, 2, \ldots, n$.

Considering the consistency of RPRs, Tanino [40] introduced the following multiplicative consistency concept.

Definition 2. [40]**.** A RPR $R = (r_{ij})_{n\times n}$ is multiplicatively consistent if the following is true:

$$r_{ij}r_{ki}r_{jk} = r_{ji}r_{ik}r_{kj} \tag{1}$$

for all $i, k, j = 1, 2, \ldots, n$.

Property 1. A RPR $R = (r_{ij})_{n\times n}$ is multiplicatively consistent if and only if $r_{ij}r_{ki}r_{jk} = r_{ji}r_{ik}r_{kj}$ for all $i, k, j = 1, 2, \ldots, n$ with $i < k < j$.

A multiplicative consistent RPR $R = (r_{ij})_{n\times n}$ can be denoted as follows:

$$r_{ij} = \frac{w_i}{w_i + w_j} \tag{2}$$

for all $i, j = 1, 2, \ldots, n$, where $w = (w_1, w_2, \ldots, w_n)$ such that $w_i \geq 0$ for all $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$ [49].

Property 2. Let $R = (r_{ij})_{n\times n}$ be a multiplicative consistent RPR, and let $w = (w_1, w_2, \ldots, w_n)$ be a weight vector such that $w_i \geq 0$ for all $i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i = 1$. Then,

$$w_i = \frac{1}{\sum_{j=1}^{n} \frac{1}{r_{ij}} - n} \tag{3}$$

for all $i = 1, 2, \ldots, n$.

Proof. From formula (2), we have the following:

$$\frac{1}{r_{ij}} = \frac{w_i + w_j}{w_i} = 1 + \frac{w_j}{w_i}.$$

Thus, $\sum_{j=1}^{n} \frac{1}{r_{ij}} = n + \frac{1}{w_i}$, by which we derive formula (3).

When a RPR $R = (r_{ij})_{n\times n}$ is multiplicatively consistent, we can apply formula (3) to derive the priority weight vector. Note that when $r_{ij} = 0$ for some j or $r_{ij} = 1$ for all j, then formula (3) makes no sense. To address this issue, we can replace $r_{ij} = 0$ with $r_{ij} = 0.0001, r_{ij} = 0.00001, \ldots$, and apply $r_{ij} = 0.9999, r_{ij} = 0.99999, \ldots$ instead of $r_{ij} = 1$. □

Property 3. Let $R = (r_{ij})_{n\times n}$ be a RPR, if its elements satisfy $r_{ij} = \frac{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}}}{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}} + \sqrt[n]{\prod_{l=1}^{n} (1-r_{il})(1-r_{lj})}}$ for all $i, k, j = 1, 2, \ldots, n$ with $i < k < j$, then R is multiplicatively consistent.

Proof. For each triple of (i, k, j), from $r_{ij} = \frac{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}}}{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}} + \sqrt[n]{\prod_{l=1}^{n} (1-r_{il})(1-r_{lj})}}$ we have

$$r_{ij}r_{ki}r_{jk} = \frac{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}}}{\sqrt[n]{\prod_{l=1}^{n} r_{il}r_{lj}} + \sqrt[n]{\prod_{l=1}^{n} (1-r_{il})(1-r_{lj})}} \times \frac{\sqrt[n]{\prod_{l=1}^{n} r_{kl}r_{li}}}{\sqrt[n]{\prod_{l=1}^{n} r_{kl}r_{li}} + \sqrt[n]{\prod_{l=1}^{n} (1-r_{kl})(1-r_{li})}}$$