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A hybrid approach to constrained global optimization☆

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ABSTRACT

In this paper, we propose a novel hybrid global optimization method to solve constrained optimization problems. An exact penalty function is first applied to approximate the original constrained optimization problem by a sequence of optimization problems with bound constraints. To solve each of these box constrained optimization problems, two hybrid methods are introduced, where two different strategies are used to combine limited memory BFGS (L-BFGS) with Greedy Diffusion Search (GDS). The convergence issue of the two hybrid methods is addressed. To evaluate the effectiveness of the proposed algorithm, 18 box constrained and 4 general constrained problems from the literature are tested. Numerical results obtained show that our proposed hybrid algorithm is more effective in obtaining more accurate solutions than those compared to.

1. Introduction

Many practical problems can be formulated as optimization problems [1,2]. A general optimization problem can be stated as follows:

$$\min \quad f(x) \tag{1}$$

$$\text{s.t.} \quad h_i(x) = 0, \quad i = 1, \ldots, l, \tag{2}$$

$$g_j(x) \le 0, \quad j = 1, \ldots, m, \tag{3}$$

$$x \in X = \{x \in \mathbb{R}^n : L \le x \le U\}, \tag{4}$$

where $x \in \mathbb{R}^n$, f, h_i, $i = 1, \ldots, l$, and g_j, $j = 1, \ldots, m$, are continuously differentiable functions, $L = [L_1, L_2, \ldots, L_n]$ and $U = [U_1, U_2, \ldots, U_n]$ are, respectively, the lower and upper bounds. Let this problem be referred to as Problem (P). To proceed further, we suppose that this problem has at least one feasible solution.

In many real world applications, Problem (P) is non-convex, either due to the non-convexity of the objective function or the constraint functions. For such a case, Problem (P) may admit many local minima. In practice, local minima are useless if their corresponding objective function values are far away from the global minimum [3]. Thus, it is important to develop methods for finding a global minimum of Problem (P). In the past several decades, there have been extensive efforts dedicated to global optimization. In general, global optimization methods can be classified into three main categories: deterministic methods, stochastic methods and hybrid methods. For the methods belonging to the first category, they are developed based on deterministic search strategies in which only deterministic information is involved for both local and global searches. In particular, for each of these methods, it relies heavily on the construction of an auxiliary function to escape from local minima, such as tunnelling function [4] and filled function [5,6], where there are several parameters to be adjusted. Tuning these parameters is computationally expensive. For the methods belonging to the second category, probabilistic techniques are utilized to escape from local minima, such as Genetic Algorithm [7–9], Ant Colony Optimization [10,11], Simulated Annealing algorithm [12], Artificial Bee Colony algorithm [13–15], Particle Swarm Optimization [16,17], Collective neuro-dynamic optimization [18], Artificial algae algorithm [19] and Differential search algorithm [20,21]. However, these methods tend to obtain solution with low accuracy and are computationally expensive due to lack of guidance by gradient during the searching process [22]. Their performances are poor in terms of convergence [23].

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The methods belonging to the third category are known as hybrid methods, where some stochastic schemes are combined together or population based search methods are combined with deterministic methods so as to speed up convergence process. In [24], Harmony Search (HS) and Artificial Bee Colony (ABC) algorithm are combined together to solve a class of box-constrained optimization problems in which ABC is incorporated to improve the local convergence of HS. In [23], a hybrid optimization technique is proposed through combining a genetic algorithm with a local search strategy based on the interior point method. In [25], an improved genetic algorithm (IGA) and an improved particle swarm optimization (IPSO) algorithm are combined and applied to optimize the amplitude of the current excitation of the spherical conformal array. In [26,3], Simulated Annealing method is used to escape from local optima obtained by gradient-based deterministic method. In [27], ABC algorithm is combined with a modified pattern search method to improve success rate and solution accuracy for box constrained optimization problems. In [28], Particle Swarm Optimizer is combined with BFGS to solve box-constrained optimization problems, where BFGS is for the local search. In [22], this hybrid method is further developed to solve general constrained optimization problems. In [29], evolutionary computation (EC) algorithms are combined with a sequential quadratic programming (SQP) algorithm to solve constrained global optimization problems. The hybrid methods mentioned above have better numerical performances when compared with pure stochastic search methods. In these hybrid methods, the stochastic methods are mainly utilized to help obtain a better initial condition for further local minimizing which means that only exploration search is used. Note that the original stochastic methods are designed not only for exploration search, but also for exploitation search. For a hybrid algorithm, if a gradient-based method is embedded for local search, the exploration would be strengthened at the expense of weakening exploitation. However, the performance of these algorithms depends heavily on tuning parameters in the stochastic algorithms. If the parameters are not tuned appropriately, the solution obtained will still be trapped into local minimum. To overcome this drawback, Dynamically Dimensioned Search Algorithm is developed in [30] where no parameters tuning is required. However, that method is a single-solution based heuristic global search algorithm. Stochastic based search methods are applicable only to unconstrained or box-constrained optimization methods, and hence are not directly applicable to solve Problem (P) which is a constrained optimization problem involving both equality and inequality constraints. In the literature, a constrained optimization problem is often transformed into a box constrained optimization problem by augmenting the constraint functions to the cost function using the augmented Lagrangian penalty method [22]. However, the penalty parameter is required to go to infinity for achieving feasibility. In this paper, the exact penalty function method (EPM) (see [31,32]) will be applied to convert the constrained optimization problem (P) into a box constrained optimization problem. A major advantage of this approach is that the penalty parameter needs only to be greater than or equal to some finite value for achieving feasibility. Then, a new population-based stochastic search method, called the Greedy Diffusion Search (GDS), is proposed to solve the box constrained optimization problems where two parameters are included. In our extensive experimental experiences, both of the two parameters can be pre-set without affecting performance and thus, no parameters tuning is required in GDS. In addition, the convergence issue is addressed. However, this method is strong in exploration but suffers from poor exploitation. Thus, the limited memory BFGS is embedded into GDS in two different strategies to improve its exploitation. An effective new hybrid search method is thus obtained for solving Problem (P).

The rest of this paper is organized as follows. In Section 2, an exact penalty method is introduced to tackle the constraints. In Section 3, two hybrid methods are proposed. Numerical results and comparisons between different methods are reported in Section 4. Section 5 concludes the paper.

2. Exact penalty function method (EPM)

Nonlinear constrained optimization problems can be solved through solving a sequence of box-constrained optimization problems by augmenting the constraint functions to the objective function using the penalty function method [6,33], to form an augmented objective function. For optimization problems with equality and inequality constraints, the penalty parameter in the augmented objective function is, in principle, required to go to infinity for achieving feasibility of the solution obtained. However, this is clearly undoable. On the other hand, the exact penalty function method introduced in [31,32] does not require the penalty parameter to go to infinity [31,32] for achieving feasibility of the solution obtained. In what follows, the exact penalty function approach proposed in [31] will be briefly described.

Let us first define the constraint violation function on X as follows:

$$G(x) = \sum_{i=1}^{l}[h_i(x)]^2 + \sum_{j=1}^{m}[\max\{g_j(x), 0\}]^2. \tag{5}$$

It is clear that $G(x)=0$ if and only if x satisfies the equality constraints (2) and the inequality constraints (3). Furthermore, $G(x)$ is a continuously differentiable function [31].

For a given $\bar{\epsilon} > 0$, we define the following penalty function on $X \times [0, \bar{\epsilon}]$:

$$F_\sigma(x, \epsilon) = \begin{cases} f(x), & \text{if } \epsilon = 0, G(x) = 0; \\ f(x) + \epsilon^{-\alpha}G(x) + \sigma\epsilon^{\beta}, & \text{if } \epsilon \in (0, \bar{\epsilon}]; \\ \infty, & \text{if } \epsilon = 0, G(x) \neq 0; \end{cases} \tag{6}$$

where $\sigma > 0$ is a penalty parameter, α and β are two positive constants satisfying $1 \le \beta \le \alpha$.

Instead of solving Problem (P) directly, let us consider the following optimization problem:

$$\min_{(x,\epsilon)\in X\times[0,\bar{\epsilon}]} F_\sigma(x, \epsilon). \tag{7}$$

Let this problem be referred to as Problem (P_σ). For a given σ, minimizing $F_\sigma(x, \epsilon)$ with respect to $(x, \epsilon) \in X \times [0, \bar{\epsilon}]$ is equivalent to minimizing $f(x) + \epsilon^{-\alpha}G(x) + \sigma\epsilon^{\beta}$. Thus, if σ is increased, ϵ^{β} will be decreased. Hence, the constraint violation $G(x)$ will be decreased. Therefore, the increase of the penalty parameter σ will eventually yield a feasible solution.

The two theorems in Appendix A reveal the relationship between Problem (P) and Problem (P_σ).

Theorem 2 in Appendix A shows that there exists a threshold $\bar{\sigma}$, such that for all $\sigma \ge \bar{\sigma}$, any local solution of Problem (P_σ) is also a local solution of Problem (P). This important property is not shared by the augmented Lagrangian penalty method [28], for which the penalty parameter is, in principle, required to go to infinity ensuring feasibility of the solution obtained. Since global solutions are included in local solutions, a global solution of Problem (P_{σ_k}) will yield a global solution of Problem (P). From this observation together with Theorems 1 and 2 in Appendix A, the exact penalty method (EPM) is utilized to convert Problem (P) into Problem (P_σ). In Section 3, an algorithm is proposed to solve Problem (P) through solving a sequence of Problem (P_σ). This algorithm is referred to as Algorithm 1.