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Hedging an option portfolio with minimum transaction lots: A fuzzy goal programming problem

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ABSTRACT

Options are designed to hedge against risks to their underlying assets such as stocks. One method of forming option-hedging portfolios is using stochastic programming models. Stochastic programming models depend heavily on scenario generation, a challenging task. Another method is neutralizing the Greek risks derived from the Black–Scholes formula for pricing options. The formula expresses the option price as a function of the stock price, strike price, volatility, risk-free interest rate, and time to maturity. Greek risks are the derivatives of the option price with respect to these variables. Hedging Greek risks requires no human intervention for generating scenarios. Linear programming models have been proposed for constructing option portfolios with neutralized risks and maximized investment profit. However, problems with these models exist. First, feasible solutions that can perfectly neutralize the Greek risks might not exist. Second, models that involve multiple assets and their derivatives were incorrectly formulated. Finally, these models lack practicability because they consider no minimum transaction lots. Considering minimum transaction lots can exacerbate the infeasibility problem. These problems must be resolved before option hedging models can be applied further. This study presents a revised linear programming model for option portfolios with multiple underlying assets, and extends the model by incorporating it with a fuzzy goal programming method for considering minimum transaction lots. Numerical examples show that current models failed to obtain feasible solutions when minimum transaction lots were considered. By contrast, while the proposed model solved the problems efficiently.

1. Introduction

An option is a contract that gives the owner of the option the right to buy or sell an underlying asset at a specified strike price on or before a specified date. The seller of that option has the obligation to fulfill the transaction when the owner chooses to exercise the option. For preventing confusions, the terms buy and sell are avoided in option trades. An option that renders the owner the right to buy at a specific price is referred to as a *call option*; conversely, an option that renders the owner the right to sell at a specific price is referred to as a *put option*. The buying of an option is referred to as a *long (long position)*, and the selling of an option is referred to as a *short (or short position)*. Normally, a call option would be exercised when the strike price is below the market value of the underlying asset, while a put option would be exercised when the strike price is above the market value. When the option expiration date passes without the option being exercised, the buyer would loss to the seller the premium (price) paid for the option. Therefore, the price of an option, and thus the pricing (valuation) of an option, holds the key to option trading strategies.

The most common option styles are American option and European option. American options may be exercised on any trading day before expiration, whereas European option can only be exercised on expiry. The market price of an American option normally closely follows the difference between the market price of the stock and the strike price of the option. However, the market price of a European option is not so forthright. The valuation of European options is a topic of ongoing research in academic and practical finance. The estimated option price is referred to as the theoretical price to contrast with the market price. Although options valuation has been studied at least since the nineteenth century, the contemporary approach is based on the Black–Scholes formula [1]

Options are originally designed to help investors hedge against risks to their underlying assets such as stocks. Conventional option-hedging strategies, such as bullish, butterfly, straddle, and spread, are determined on the basis of an investor's estimate of the future

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Nomenclature

k	Option strike price
r	Risk-free interest rate
s	Stock price
σ	Stock volatility
t	Current time
T	Time to maturity
$N()$	The standard normal cumulative distribution function
δ	Option delta
γ	Option gamma
θ	Option theta
ρ	Option rho
κ	Option kappa
c	Call option
p	Put option
m	Number of option
n	Number of stock
v	Option theoretical price
v'	Option market price
R	Total profit
u	Long stock position
x	Long option position
y	Short option position
z	Short stock position
f	Option transaction cost
V	Option portfolio value
Γ	Portfolio gamma
Δ	Portfolio delta
Θ	Portfolio theta
K	Portfolio kappa
P	Portfolio rho
Φ	Portfolio scale limit
g^*	Aspired goal value for objective g
g^+	Maximum tolerance limit for objective g
g^-	Minimum tolerance limit for objective g
μ_g	Membership function of objective g
w_g	Weight of objective g
λ	Minimum aspiration level among objectives

prices of the underlying assets. For example, when the trader expects the underlying stock price to move upwards, a bullish strategy can be implemented by buying a call option of low strike price while simultaneously selling a call option of a high strike price, both of the same underlying security and the same expiration month. These strategies are not completely risk free, especially when the future price movement is contrary to the investor's expectations. Furthermore, the strategies are difficult to implement when multiple assets are involved.

Regarding hedging risks, a portfolio with multiple options appears more attractive and effective than that with only a single option. One method of forming option-hedging portfolios is using stochastic programming models based on various scenario generation methods. At every node of a scenario tree, a decision-maker determines possible ex post prices for the target asset (for example, a stock). Leading to another node, each branch from a node represents a price scenario associated with an occurrence probability. Owing to the powers of current computers, stochastic programming has recently gained popularity in the mathematical programming community. For example, Wu and Sen [2] developed a stochastic programming model for currency option hedging. This model replicates a target option by constructing a portfolio of options and futures. The objective function minimizes a combination of four terms: the expected shortfall of payoff tracking error, the expected delta tracking error, the expected gamma tracking error, and the expected cost for setting up the tracking portfolio. Topaloglou et al. [3] considered a problem of controlling the risk exposure of a portfolio consisting of options and forwards among multiple markets. They proposed a one-stage stochastic programming model that minimized the portfolio conditional value-at-risk (CVaR) [4] under an expected return constraint. On the basis of the work by Papahristodoulou [5], Yin and Han [6] proposed a multistage stochastic programming model for optimizing an international option portfolio that involved options of multiple underlying assets among different markets. The model minimizes the CVaR of a portfolio return in a manner similar to the model by Topaloglou et al. [3].

However, stochastic programming models depend heavily on scenario generation. Yin and Han [7] indicated that the performance of a stochastic optimization model greatly depends on the quality of a scenario tree. Otherwise, the stochastic optimization technique is merely a theoretical fantasy rather than a practicable tool. Therefore, studies have been devoted to scenario generation. Kaut and Wallace [8] formulated minimal requirements that should be imposed on a scenario generation method before it can be used for solving a stochastic programming model. They also showed how to test the requirements. Davari-Ardakani et al. [9] developed a new scenario generation method that characterizes the dynamic behavior of asset returns and considers the dependence structure of different asset returns and the serial return correlations of each asset.

Another method for hedging options is to neutralize the risks associated with the Greek letters (delta, gamma, theta, rho, and kappa) derived from the Black–Scholes formula [1]. Neutralizing the Greek risks involves constructing a portfolio in which the Greeks remain zero when any of the underlying variables change within a small range. A portfolio consisting of underlying assets and corresponding options can also provide arbitrage profits. Linear programs have been proposed for devising hedge strategies for option portfolios. Rendleman [10] might have been the first to propose a linear programming model for maximizing the difference between the theoretical and market price of options of a single asset subject to constraints that neutralize some of the Greek risks. By considering all of the Greek risks, Papahristodoulou [5] proposed a model similar to that by Rendleman [10]. Horasanli [11] extended Papahristodoulou's model to multiple underlying assets.

Hedging Greek risks requires no human intervention for generating scenarios and associated occurrence probabilities; however, the current models are not without problems. One problem with the models is that they require that the Greek risks be perfectly neutralized, leading to many equality constraints. Feasible solutions that conform to these equality constraints might not exist. Gao [12] attempted to resolve this problem by relaxing the constraints pertaining to risk neutralization. However, selecting the allowable risk bounds in Gao's model is arbitrary such that the model can still be infeasible. Furthermore, the models can hedge only short-term risks because the Greek risks tend to change with time. Long-term hedging is possible but again requires scenario generation and an optimization method such as stochastic programming. Yin and Han [7], for example, proposed a multistage stochastic programming model for option strategies that incorporated the method by Gao [12].

Another problem is that the current models that involve multiple assets and their derivatives, including those of Yin and Han [6], Gao [12], and Yin and Han [7], were incorrectly formulated. When an option portfolio involves multiple underlying assets with independent prices and volatilities, some Greek risks (such as the deltas) with respect to different underlying assets can act differently. Con-